KNL Chanj

ELECTRIC AND MAGNETIC FIELDS

UNIT-I

ELECTROSTATICS - I

Coordinate Systems - Introduction to Coordinate systems, Rectangular, Cylindrical and Spherical Coordinate systems. Coulomb's Law and Electrostatic Field Intensity - Coulomb's Law, Electric field, Electric Field Intensity (EFI), Electric fields due to Continuous charge distributions - Volume charge, surface charge, line charge.
EFI due to a line and a surface charge.
Electric Flux density and Gauss's Law : Electric flux, Electric flux density, Gauss's law, Applications of Gauss's law, Maxwell's equation.

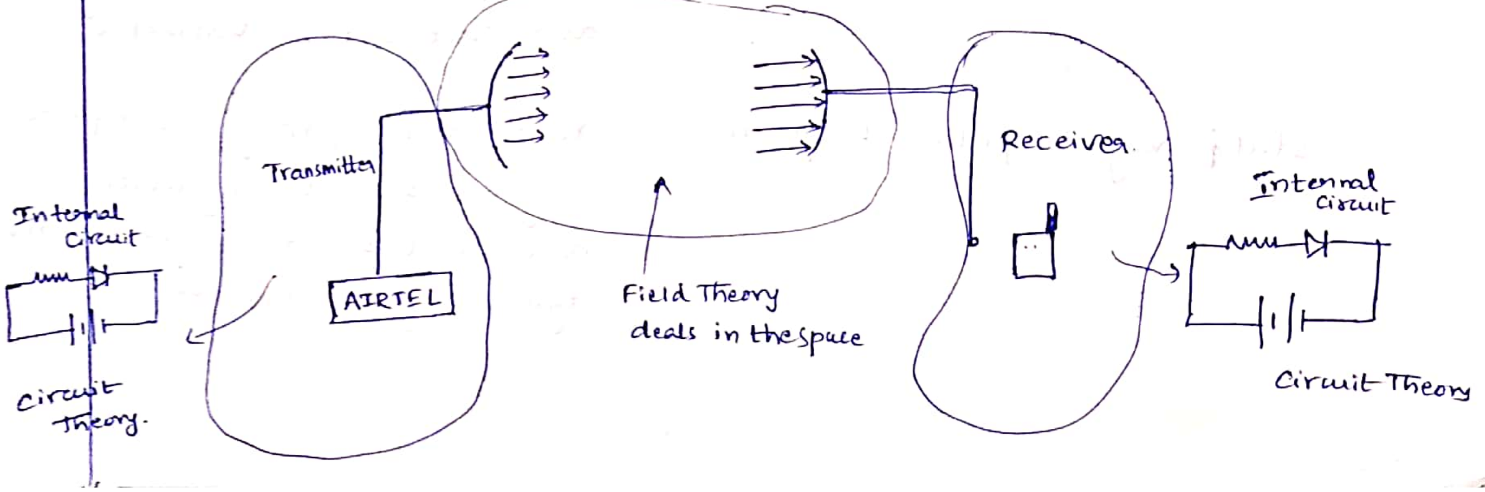
Introduction:

→ Electromagnetic Theory is concerned with the study of charges at rest and in motion.

→ Electromagnetic principles are fundamental to the study of electrical engineering.

→ Electromagnetic theory is also required for the understanding, analysis and design of various electrical, electromechanical and electronic systems.

"Electromagnetics is a branch of physics or electrical engineering in which electric and magnetic phenomena are studied".



→ In the space, Circuit theory is failed because all the Components are distributed.

→ In Communication systems, circuit theory is valid at both the transmitting end as well as the receiving end but it fails to explain the flow between the transmitter and receiver.

→ Circuit theory deals with only two variables i.e., Voltage & Current whereas Electromagnetic theory deals with many Variables electric-field intensity, magnetic field intensity etc.,

→ Electromagnetic theory can be thought of as generalization of Circuit theory. Electromagnetic theory deals directly with electric and magnetic field vectors where as circuit theory deals with Voltages and currents.

Voltages and Currents are integrated effects of electric and magnetic fields respectively.

→ Electromagnetic field problems involves three space Variables along with the time Variable and hence the solution becomes Complex. So, we need strong background of Vector analysis.

Applications of Electromagnetic theory:

This subject basically Consists of static electric fields, static magnetic fields, time-varying fields & its applications

Electrostatic fields — used in — CRO's, ink-jet printer, Ore seperation, electrostatic generator and electrostatic Voltmeter

Static magnetic fields used in — dc machines, magnetic deflection, magnetic seperator, cyclotron, hall effect Sensors, magneto hydro-dynamic generator etc.

Vector Analysis :

"Vector Analysis is a mathematical tool with which Electromagnet Concepts are most Conveniently expressed and best Comprehended."

→ A quantity can be either a scalar or a vector.

Scalar :

"A Scalar is a physical quantity that has only magnitude"

Ex: speed, mass, temperature, volume, density, electric charge.

Vector :

"A vector is a physical quantity that has both magnitude and direction".

Ex: Force, Velocity, Acceleration, EFI & MFI.

Scalar field:

"The distribution of a scalar quantity with a definite position in a space is called scalar field".

Ex: Temperature of atmosphere which has a definite value but no need of direction to specify it.

Vector field:

"The distribution of vector quantity in a space is called Vector field".

Ex: The gravitational force on a mass in a space. This force has a value at various points in a space and also has a specific direction.

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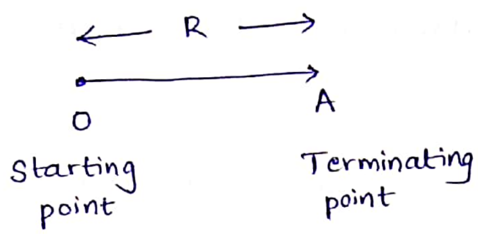
Representation of a Vector:

→ Vector can be represented by a straight line with an arrow in a planor surface.



$\Rightarrow \overline{OA}$

The length of the segment is called magnitude and arrow indicates direction.

$$\overline{OA} = |\overline{OA}|\ \hat{a}_{OA}$$

Where

$|\overline{OA}|$ = Magnitude of $\overline{OA}$ = R

$\hat{a}_{OA}$ = Unit vector along the direction of $\overline{OA}$

Unit Vector:

Its magntuide always one. i.e., $|\hat{a}_{OA}| = 1$

$$\hat{a}_{OA} = \frac{\overline{OA}}{|\overline{OA}|}$$

(P) Obtain the unit Vector in the direction from the Origin towards the point P(3,-3,2).

Sol:

$$\hat{a}_{op} = \frac{\overline{OP}}{|\overline{OP}|}$$

$$= \frac{3\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z}{\sqrt{3^2+(-3)^2+2^2}} = \frac{3\bar{a}_x - 3\bar{a}_y + 2\bar{a}_z}{4.6904}$$

$$= 0.6396\,\bar{a}_x - 0.6396\,\bar{a}_y + 0.4264\,\bar{a}_z$$

(P) Two points A(2,2,1) & B(3,−4,2) are given in the Cartesian system. Obtain the vector from A to B & unit vector directed from A to B.

Sol:

$$\hat{a}_{AB} = \frac{\overline{AB}}{|\overline{AB}|}$$

$$\overline{AB} = (x_2 - x_1)\,\bar{a}_x + (y_2 - y_1)\,\bar{a}_y + (z_2 - z_1)\,\bar{a}_z$$

$$= \bar{a}_x - 6\,\bar{a}_y + \bar{a}_z$$

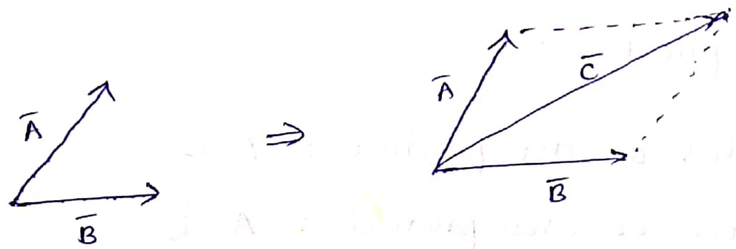
$$|\overline{AB}| = \sqrt{1^2 + (-6)^2 + (1)^2} = \sqrt{38} = 6.1644$$

$$\hat{a}_{AB} = \frac{\overline{AB}}{|\overline{AB}|} = \frac{\bar{a}_x - 6\bar{a}_y + \bar{a}_z}{6.1644}$$

$$= 0.1622\,\bar{a}_x - 0.9733\,\bar{a}_y + 0.1622\,\bar{a}_z.$$

Addition of two Vectors :

parallelogram law



$$\therefore \bar{C} = \bar{A} + \bar{B}$$

Position and Distance Vectors :

The position vector r_p (or radius vector) of point P is defined as the directed distance from the Origin O to P

i.e., $r_p = OP = x\bar{a}_x + y\,\bar{a}_y + z\cdot\bar{a}_z$

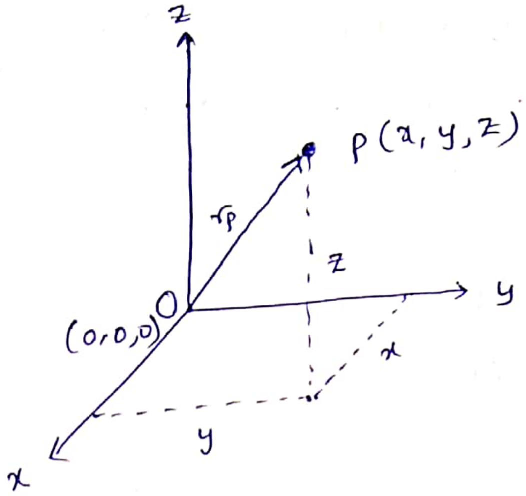
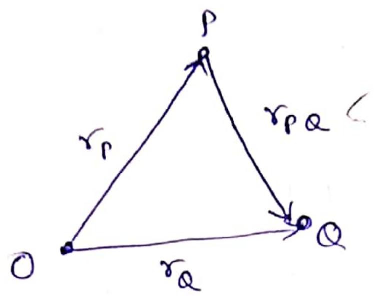


Fig: Illustration of position vector r_P.

"The distance vector is the displacement from one point to another".



$$r_{PQ} = r_Q - r_P = (x_Q - x_P)\bar{a}_x + (y_Q - y_P)\bar{a}_y + (z_Q - z_P)\bar{a}_z$$

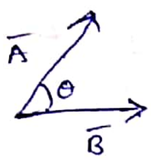
Vector Multiplication :-

(1) Scalar or dot product : $\bar{A}\cdot\bar{B}$

(2) Vector or Cross product : $\bar{A}\times\bar{B}$

Scalar or dot product :

The dot product of two vectors A & B is defined geometrically as the product of the magnitudes of A & B and the cosine of the smaller angle between them.



$$\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|\cos\theta$$

$$\cos\theta = \frac{\bar{A}\,\bar{B}}{|\bar{A}||\bar{B}|}$$

$$\therefore\ \theta = \cos^{-1}\left[\frac{\bar{A}\,\bar{B}}{|\bar{A}||\bar{B}|}\right]$$

→ Dot product of two vectors gives the scalar.

properties :

(1) If two vectors are parallel to each other, then $\theta = 0$,

$$\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|\cdot\cos\theta$$

$$= |\bar{A}|\,|\bar{B}|\cdot\cos 0$$

$$= |\bar{A}|\,|\bar{B}|$$

(2) If two vectors are perpendicular each other, then $\theta = 90^\circ$

$$\bar{A}\cdot\bar{B} = |\bar{A}|\,|\bar{B}|\cos\theta$$

$$= |\bar{A}|\,|\bar{B}|\cos 90^\circ$$

$$= 0$$

(3) Dot product obeys Commutative law.

$$\bar{A}\cdot\bar{B} = \bar{B}\cdot\bar{A}$$

(4) Dot product obeys distributive law.

$$\bar{A}\cdot(\bar{B}+\bar{C}) = \bar{A}\cdot\bar{B} + \bar{A}\cdot\bar{C}$$

$$\bar{A}\cdot\bar{A} = |A|^2 = A^2$$

(5) Also note that

$$\bar{a}_x\cdot\bar{a}_y = \bar{a}_y\cdot\bar{a}_z = \bar{a}_z\cdot\bar{a}_x = 0$$

$$\bar{a}_x\cdot\bar{a}_x = \bar{a}_y\cdot\bar{a}_y = \bar{a}_z\cdot\bar{a}_z = 1$$

NOTE

$\bar{a}_x, \bar{a}_y$ & $\bar{a}_z$ are the base vectors along x, y & z axis respectively.

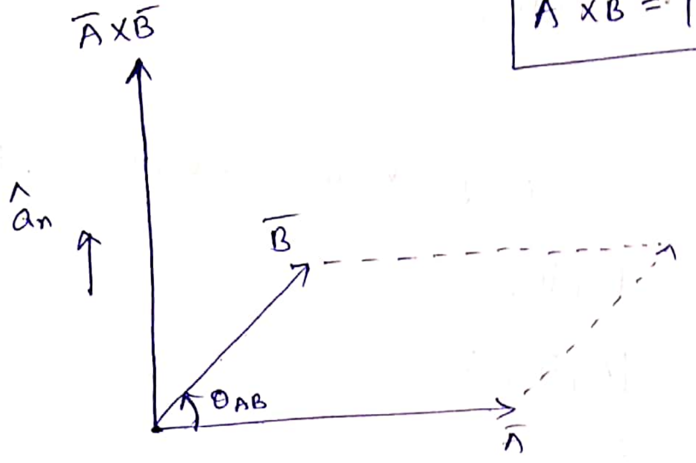
(or)

$\hat{i}, \hat{j}, \hat{k}$

Cross product :

The Cross product of two vectors $\bar{A}$ and B is a vector quantity whose magnitude is the area of the parallelogram formed by A and B and the direction is perpendicular to the plane containing the vectors A and B.

$$\boxed{\bar{A} \times \bar{B} = |\bar{A}|\,|\bar{B}|\,\sin\theta_{AB}\,\hat{a}_n}$$



where θ = Angle between $\bar{A}$ and $\bar{B}$

$\hat{a}_n$ = Unit vector normal to the plane containing $\bar{A}$ & $\bar{B}$.

properties :

Anti-Commutative

(1) It is not commutative i.e., $\bar{A}\times\bar{B} \neq \bar{B}\times\bar{A}$ & $\bar{A}\times\bar{B} = -\bar{B}\times\bar{A}$
$\bar{B}\times\bar{A} = -\bar{A}\times\bar{B}$

(2) It is not associative i.e., $\bar{A}\times(\bar{B}\times\bar{C}) \neq (\bar{A}\times\bar{B})\times\bar{C}$

(3) It is distributive i.e., $A\times(B+C) = (A\times B) + (A\times C)$

(4) If two vectors are parallel, $\theta = 0$, $\sin\theta = 0$

$$\therefore \bar{A}\times\bar{B} = 0$$

(5) If two vectors are perpendicular, $\theta = 90^\circ$, $\sin 90^\circ = 1$

$$\bar{A}\times\bar{B} = |\bar{A}|\,|\bar{B}|\cdot\hat{a}_n$$

(6) $\bar{A}\times\bar{A} = 0$

(7) $\bar{a}_x\times\bar{a}_y = \bar{a}_z$; $\bar{a}_y\times\bar{a}_z = \bar{a}_x$; $\bar{a}_z\times\bar{a}_x = \bar{a}_y$

Operator Del (∇) :-

Del is a vector three dimensional partial differential operator.

It is defined in Cartesian system as

$$\boxed{\nabla = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}}$$

Del is very important operator. There are 3 possible opearations with del. They are gradient, divergence and curl.

(i) Gradient :

Gradient is a basic operation of a Del operator that can operate only on a scalar function.

Consider a scalar function 't',

$$\underset{(\text{Grad } t)}{\nabla t} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] t$$

$$\boxed{\nabla t = \frac{\partial t}{\partial x}\hat{i} + \frac{\partial t}{\partial y}\hat{j} + \frac{\partial t}{\partial z}\hat{k}}$$

→ Vector.

Gradient of scalar function is a vector function.

Ex: Temperature of soldering iron is scalar, but rate of change of temperature is a vector.

In a cable, potential is scalar, The rate of change of potential is a vector (Electric field intensity).

(ii) Divergence:

"Divergence is a basic operation of the Del operator which can operate only on a Vector function through a dot product."

Consider a Vector function $\bar{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

The divergence of Vector A mathematically and symbolically expressed as shown below.

$$\nabla \cdot \bar{A} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] \cdot \left[A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right]$$

(Div A)

$$\boxed{\nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

↳ Scalar

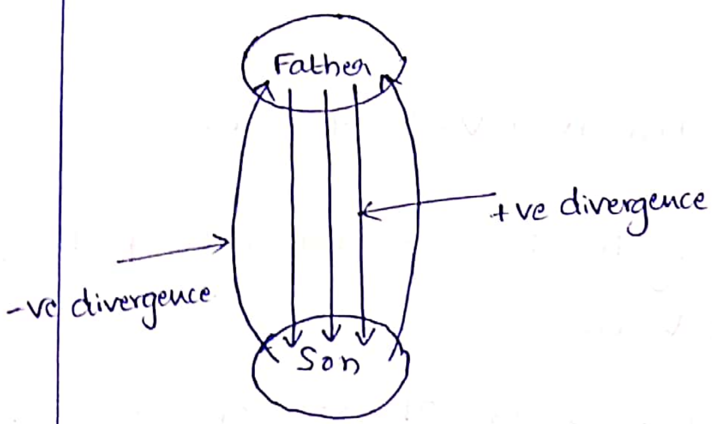
→ Divergence of vector function is a scalar function.

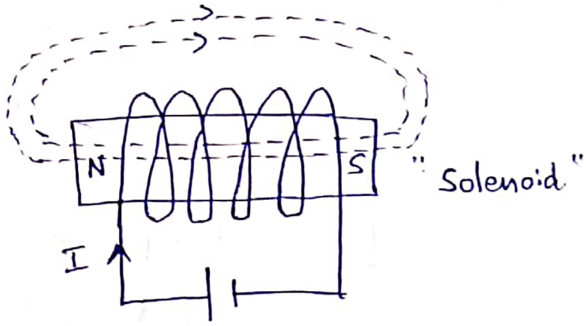
(a) +ve divergence

(b) -ve divergence

(c) Zero divergence → $\boxed{\nabla \vec{A} = 0}$

then $\vec{A}$ is solenoidal vector





→ In the solenoid, Net divergence flux is zero

$\nabla \cdot \phi = 0$.

(iii) Curl:

Curl is a basic operation of a Del operator which can perform only on a vector function through a cross product.

$$\nabla \times \vec{A} = \left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right] \times \left[A_x\hat{i} + A_y\hat{j} + A_z\hat{k}\right]$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

→ Vector

$$= \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right]\hat{i} - \left[\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right]\hat{j} + \left[\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right]\hat{k}$$

→ Curl of a Vector function is a Vector function

→ Curl deals with rotation.

→ If the Curl of a Vector field Vanishes, it is Called Irrotational field.

(a) +ve Curl

(b) −ve Curl

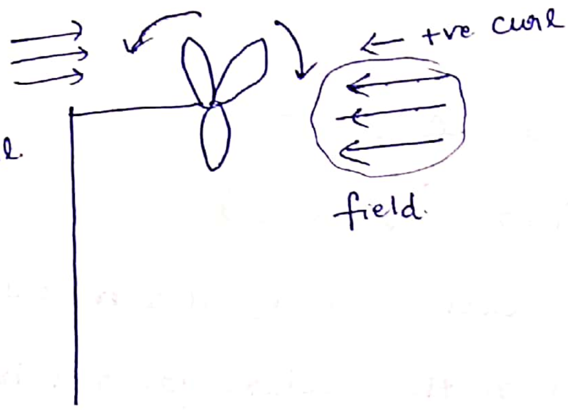
(c) Zero Curl → $\boxed{\nabla \times \vec{A} = 0}$

Then $\vec{A}$ is Called irrotational,

(or) Conservative (or) Curl free

(or) lamallar Vector.

-ve Curl



Laplacian of a Scalar function (t) :

↳ Double Operation

$$\nabla\cdot(\nabla t) = \nabla^2 t = \frac{\partial^2 t}{\partial x^2} + \frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2}$$

$$\boxed{\nabla^2 t = \underbrace{\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]}_{\text{Laplacian Operator.}} t}$$

→ Laplacian of a scalar function is a scalar function.

Laplacian of a Vector function ($\vec{A}$) :

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$$\nabla^2\vec{A} = \left[\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial^2 A_x}{\partial z^2}\right]\hat{i} + \left[\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial^2 A_y}{\partial z^2}\right]\hat{j}$$
$$+ \left[\frac{\partial^2 A_z}{\partial x^2} + \frac{\partial^2 A_z}{\partial y^2} + \frac{\partial^2 A_z}{\partial z^2}\right]\hat{k}$$

→ Laplacian of a vector function is a vector function.

Basic Types of Vector fields :

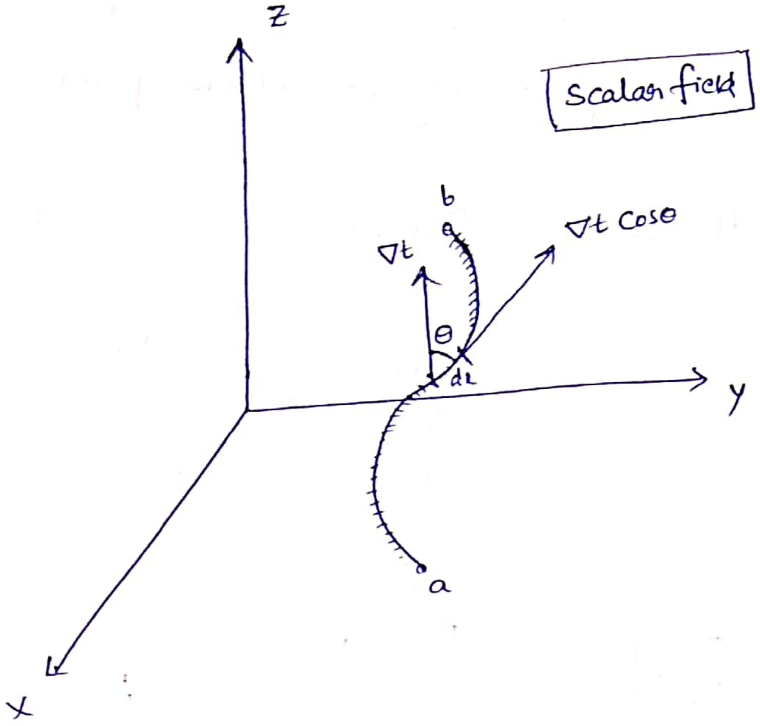
(a) Solenoidal Vector field ($\nabla\cdot\vec{A} = 0$)

(b) Irrotational Vector field ($\nabla\times\vec{A} = 0$)

(c) Vectors fields that are both Solenoidal and irrotational.

(d) Vector fields which are neither Solenoidal nor irrotational.

Fundamental Theorem of Gradient :



Statement Consider an open path from 'a' to 'b' in a scalar field. The line integral of the tangential component of the gradient of a scalar function along the open path is equal to path the effective value of the associated scalar function at the boundaries of the open path.

If 't' is the associated scalar function, then according to the fundamental theorem of gradient

$$\boxed{\int_a^b (\nabla t)\cdot \overline{dl} = t(b) - t(a)}$$

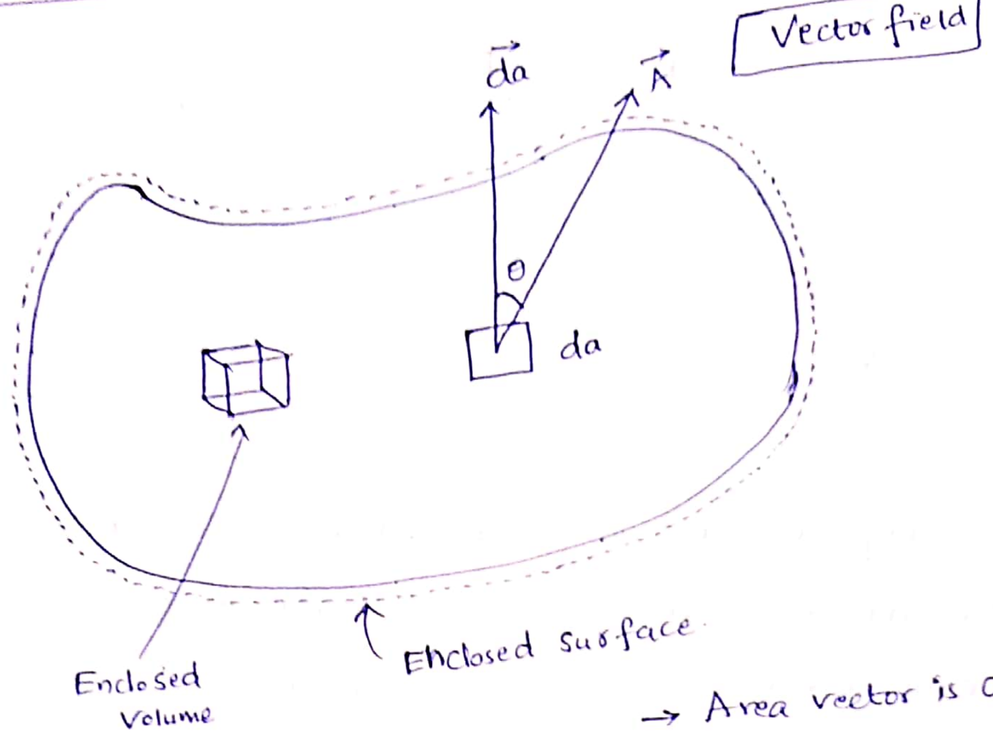
Corollary-1 If it is a closed path in scalar field, then

$$\boxed{\int_a^b (\nabla t)\cdot \overline{dl} = 0.}$$

Corollary-2:

A line integral $\int_a^b (\nabla t)\cdot \vec{dl}$ is independent of open path i.e., it depends only on extreme points.

Fundamental theorem of Divergence: (Gauss Theorem)



→ Area vector is always outward normal.

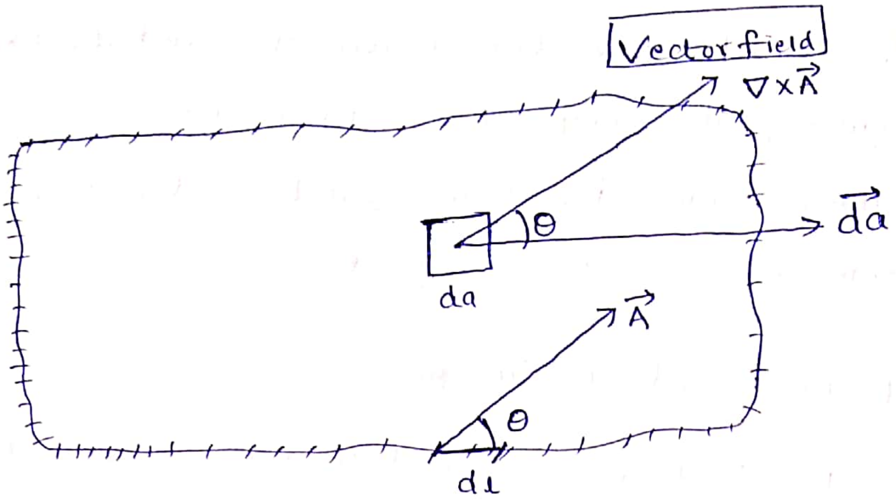
Statement Consider a closed surface in a vector field. The volume integral of the divergence of the associated vector function carried with in a ~~cl~~ enclosed volume is equal to the surface integral of the normal component of the associated vector function carried over an enclosing surface.

If associated vector function is $\vec{A}$, then according to fundamental theorem of divergence,

$$\boxed{\iiint_V (\nabla\cdot\vec{A})\, dv = \oint\!\!\oint_S \vec{A}\cdot ds}$$

i.e., The inside material is equal to outside closed surface because of divergence.

Fundamental Theorem of Curl :- (Stokes theorem).



Statement Considering a open surface placed in a vector field, the surface integral of the normal component of the curl of the associated vector function carried over the open surface is equal to the line integral of the tangential component of the associated vector function along the boundary of the open surface.

If associated vector function is $\vec{A}$, then

$$\boxed{\iint_S (\nabla \times \vec{A}) \cdot \vec{da} = \oint \vec{A} \cdot dl}$$

Corollary-1: If it is a closed surface

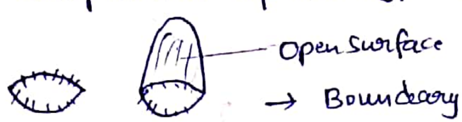
$$\boxed{\oiint (\nabla \times \vec{A}) \vec{da} = 0}$$

Since there is no boundary and hence

$$\boxed{\oint \vec{A} \cdot dl = 0}$$

Corollary-2: $\oint \vec{A} \cdot dl$ is constant for fixed boundary.

$\therefore$ $\iint_S (\nabla \times \vec{A}) \, \vec{da}$ is independent of the type of open surface.



→ Boundary is same for both but open surface is not same.

Co-ordinate Systems :

To describe a vector accurately and to express a vector interms of its Components, It is necessary to have reference some direction. Such a directions are represented interms of Co-ordinate systems.

Types of Coordinate Systems:

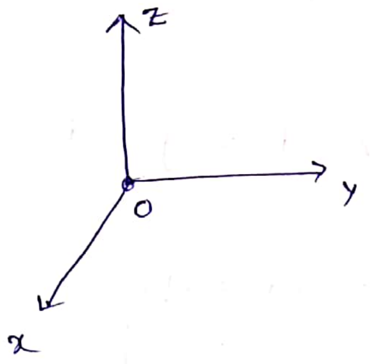
(1) Cartesian (or) rectangular Co-ordinate system

(2) Cylindrical Co-ordinate system.

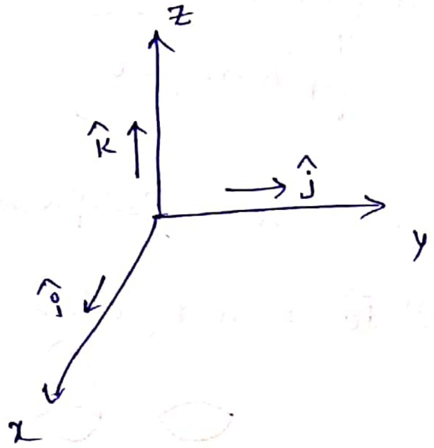
(3) Spherical Co-ordinate system.

(1) Cartesian (or) rectangular Co-ordinate System :

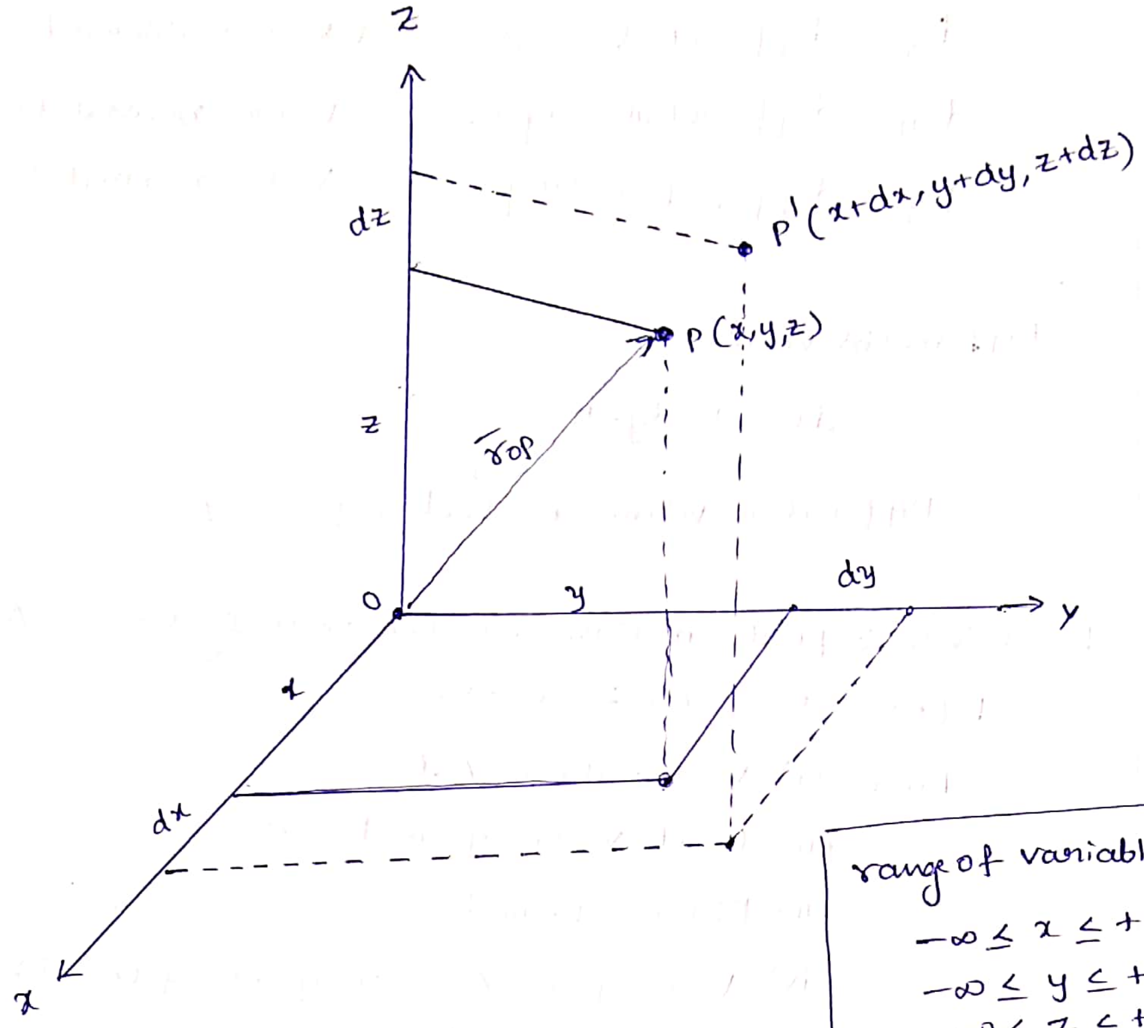
Three Coordinates are x, y, z which are mutually right angle to each other. These three axes intersect at a common point is called origin (o).



Base vectors :



$\hat{i}, \hat{j}$ & $\hat{k}$ are base vectors.



range of variables

$-\infty \leq x \leq +\infty$

$-\infty \leq y \leq +\infty$

$-\infty \leq z \leq +\infty$

$\bar{r}_{op}$ = position Vector

$= x\hat{i} + y\hat{j} + z\hat{k}$

$|\bar{r}_{op}| = \sqrt{x^2 + y^2 + z^2}$

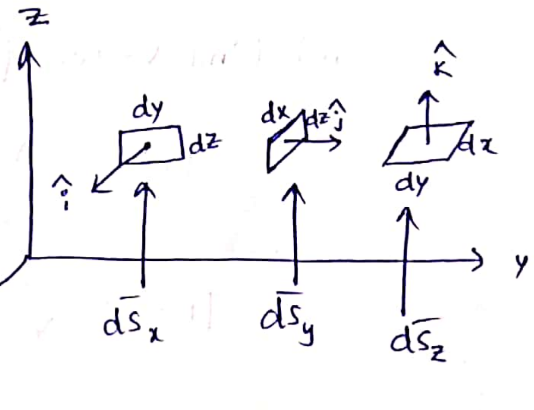
Differential ~~vector~~ length Vector, $\vec{dl} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$

Differential Surface area : $(\bar{ds})$

$\bar{ds}_x = dy \cdot dz \cdot \hat{i}$

$\bar{ds}_y = dx \cdot dz \cdot \hat{j}$

$\bar{ds}_z = dx \cdot dy \cdot \hat{k}$



$\bar{ds_x}$ = differential surface area vector normal to x-direction.

$\bar{ds_y}$ = differential surface area vector normal to y-direction.

$\bar{ds_z}$ = differential surface area vector normal to z-direction.

Differential Volume:

$$dv = dx.dy.dz$$

Differential volume is scalar quantity.

(P) Given 3 points in Cartesian coordinate system as A (3,-2,1), B(-3,-3,5) & C(2,6,-4).

Find (i) Vector from A to C

(ii) Unit vector from B to A

(iii) Distance from B to C

(iv) Vector from A to midpoint of the straight line joing B to C.

Sol:

$$\bar{A} = 3\hat{i} - 2\hat{j} + \hat{k} = 3\bar{a}_x - 2\bar{a}_y + \bar{a}_z$$

$$\bar{B} = -3\hat{i} - 3\hat{j} + 5\hat{k} = -3\bar{a}_x - 3\bar{a}_y + 5\bar{a}_z$$

$$\bar{C} = 2\hat{i} + 6\hat{j} - 4\hat{k} = 2\bar{a}_x + 6\bar{a}_y - 4\bar{a}_z$$

(i) Vector from A to C

$$\bar{AC} = \bar{C} - \bar{A}$$

$$= -\bar{a}_x + 8\bar{a}_y - 5\bar{a}_z$$

(ii) Unit Vector from B to A $(\bar{a}_{BA})$

$$\bar{BA} = \bar{A} - \bar{B}$$

$$= 6\bar{a}_x + \bar{a}_y - 4\bar{a}_z$$

$$|\bar{BA}| = \sqrt{36+1+16} = \sqrt{53}$$

$|\overline{BA}| = 7.28$

$$\bar{a}_{BA} = \frac{\overline{BA}}{|\overline{BA}|} = \frac{6\bar{a}_x + \bar{a}_y - 4\bar{a}_z}{7.28} = 0.822\,\bar{a}_x + 0.137\,\bar{a}_y - 0.549\,\bar{a}_z$$

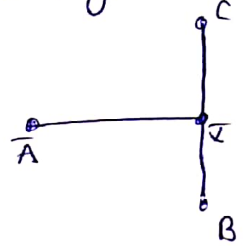
(iii) Distance from B to C.

$$\overline{BC} = \bar{C} - \bar{B}$$

$$= 5\bar{a}_x + 9\bar{a}_y - 9\bar{a}_z$$

$$|\overline{BC}| = \sqrt{25+81+81} = \sqrt{187} = 13.67$$

(iv) Vector from $\bar{A}$ to midpoint of the st. line joining B to C.



$$\bar{X} = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$

$$= -0.5\bar{a}_x + 1.5\bar{a}_y + 0.5\bar{a}_z$$

$$\overline{AX} = \bar{X} - \bar{A} = -3.5\bar{a}_x + 3.5\bar{a}_y - 0.5\bar{a}_z$$

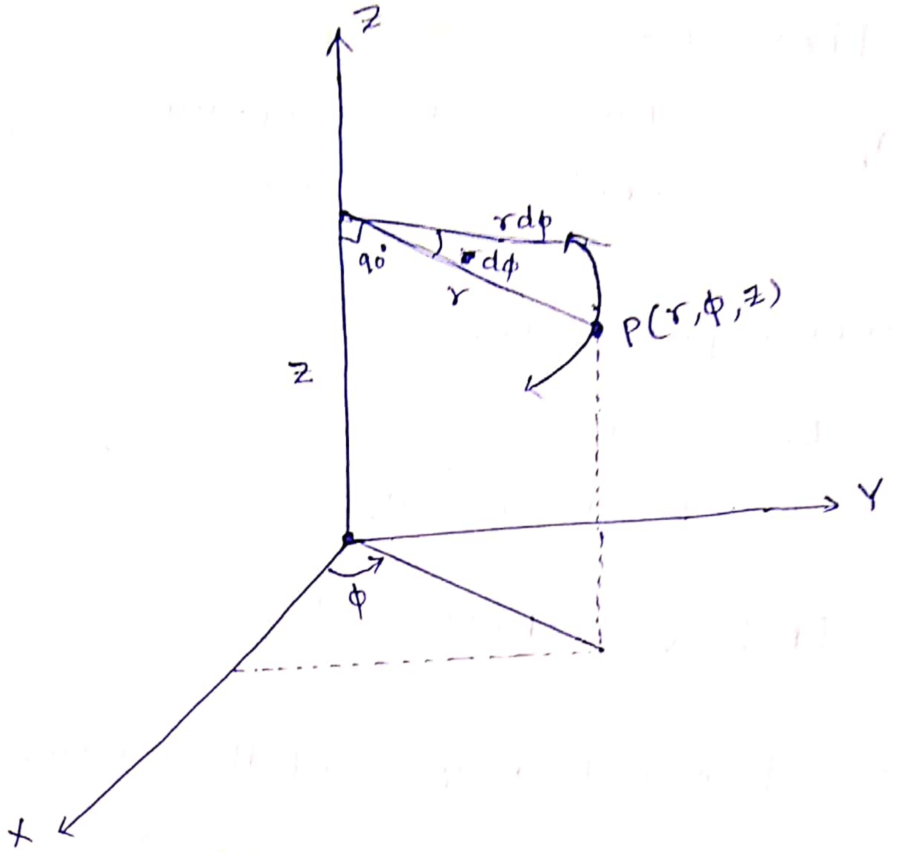
2. Cylindrical Co-ordinate System:

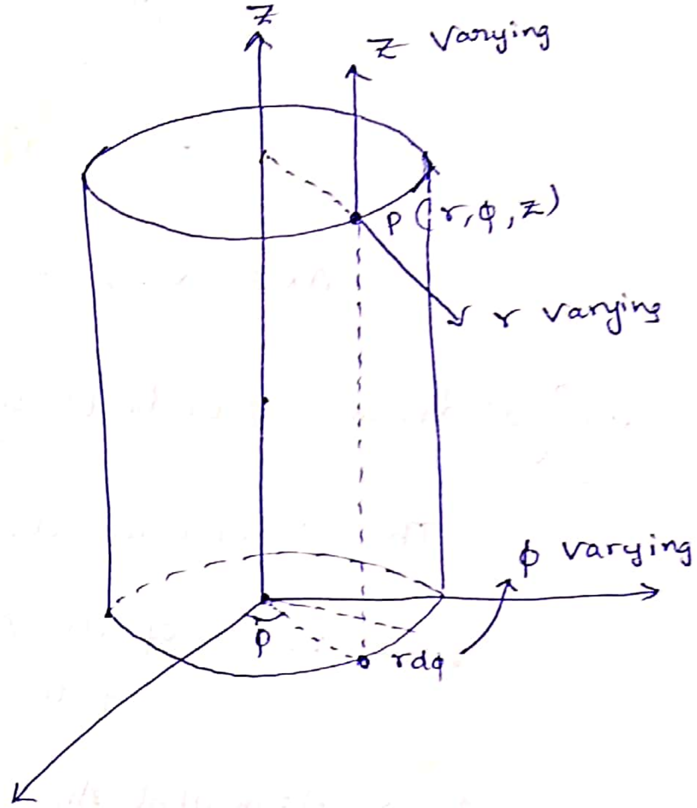
The Co-ordinates are r, ϕ, z.

$r \rightarrow$ radius of the cylinder with z-axis as the axis of the cylinder.

$\phi \rightarrow$ Azimuthal Angle, ~~A half plane~~ is measured from the x-axis in the xy-plane.

$z \rightarrow$ Same in the Cartesian system.





Range of Variables

$0 \le r \le \infty$

$0 \le \phi \le 2\pi$

$-\infty \le z \le +\infty$

Base vectors

$\bar{a}_r, \bar{a}_\phi, \bar{a}_z$

(or)

$\hat{r}, \hat{\phi}, \hat{z}$

Differential elements

$dr, r\,d\phi, dz$

Differential length Vector

$$\vec{dl} = dr\,\bar{a}_r + r.d\phi\,\bar{a}_\phi + dz\,\bar{a}_z$$

$$|\vec{dl}| = \sqrt{(dr)^2 + (r.d\phi)^2 + (dz)^2}$$

Differential Volume:

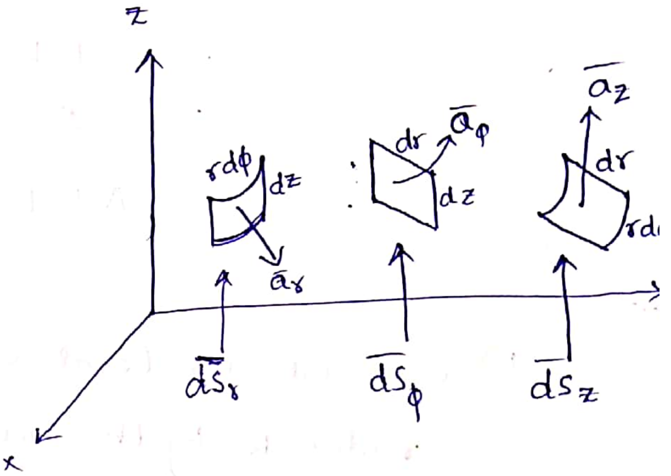
$$dv = r.dr.d\phi.dz.$$

Differential Surface Area:

$$\bar{ds}_r = r\,d\phi.dz\,\bar{a}_r$$

$$\bar{ds}_\phi = dr.dz.\bar{a}_\phi$$

$$\bar{ds}_z = r.dr.d\phi.\bar{a}_z$$



Cartesian to Cylindrical

$$r = \sqrt{x^2+y^2}$$

$$\phi = \tan^{-1}(y/x)$$

$$z = z$$

Cylindrical to Cartesian

$$x = r\cos\phi$$

$$y = r\sin\phi$$

$$z = z$$

(P) Calculate the Volume of the ~~Sphere~~ cylinder having length L and radius R.

Sol:

$$dv = r.dr.d\phi.dz$$

$$V = \int_0^L \int_0^{2\pi} \int_0^R r.dr.d\phi.dz$$

$$= \int_0^L \int_0^{2\pi} \left[\frac{r^2}{2}\right]_0^R d\phi.dz$$

$$V = \int_0^L \int_0^{2\pi} \frac{R^2}{2} \, d\phi . dz$$

$$= \frac{R^2}{2} \int_0^L [\phi]_0^{2\pi} dz$$

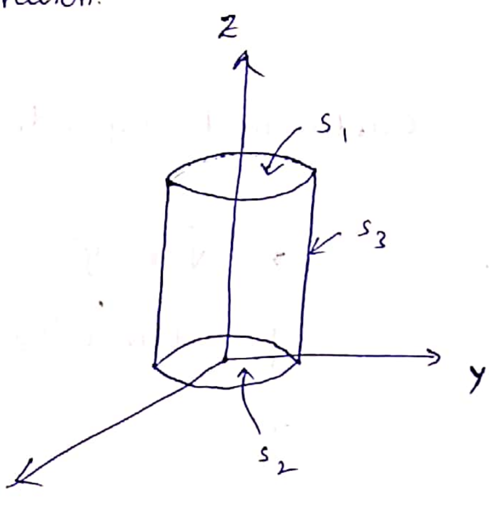
$$= \frac{R^2}{2} \int_0^L 2\pi . dz$$

$$= \frac{2\pi R^2}{2} [z]_0^L$$

$$= \frac{\cancel{2}\pi R^2 L}{\cancel{2}}$$

$$\boxed{V = \pi R^2 L}$$

(P) Calculate the total surface area of the cylinder having length L, radius R by the method of integration.



Sol:

$$\bar{ds}_z = r.dr.d\phi \; \bar{a}_z$$

$$S_1 = \int_0^{2\pi} \int_0^R r.dr.d\phi . \bar{a}_z$$

$$= \frac{R^2}{2} [\phi]_0^{2\pi} = \pi R^2$$

$$\boxed{S_1 = S_2 = \pi R^2}$$

$$S_3 = \int \bar{ds}_r = \int_0^L \int_0^{2\pi} r.d\phi.dz . \bar{a}_r \quad , \; r = R$$

$$= R . [\phi]_0^{2\pi} \int_0^L dz .$$

$$= 2\pi R [z]_0^L$$

$$= 2\pi RL$$

$$S = S_1 + S_2 + S_3$$

$$= \pi R^2 + \pi R^2 + 2\pi R L$$

$$= 2\pi R^2 + 2\pi R L$$

$$\boxed{S = 2\pi R[R+L]}$$

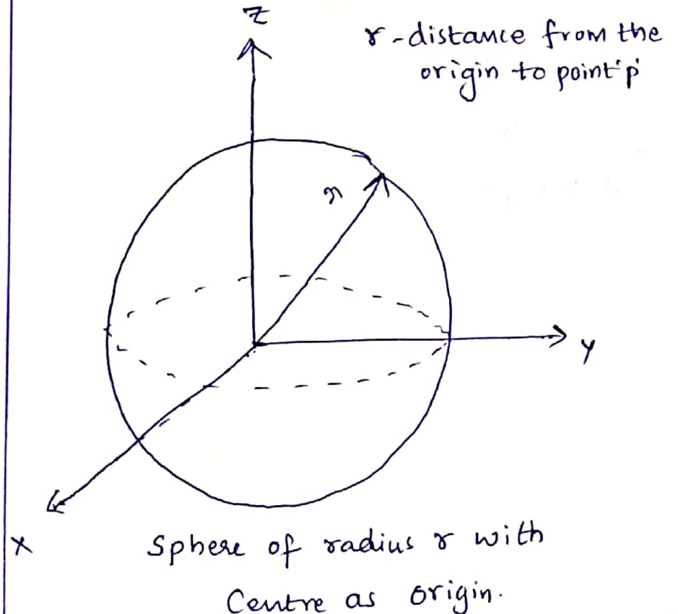
(3) Spherical Co-ordinate System:

Three Coordinates are r, θ, ϕ

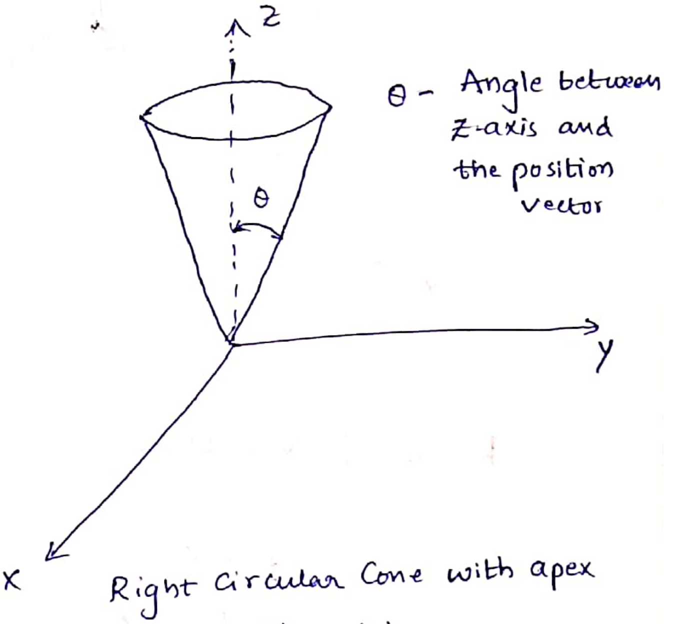
$r \rightarrow$ radius of the Sphere with Centre as origin.

$\theta \rightarrow$ Right circular Cone with apex at origin and its axis as z-axis.

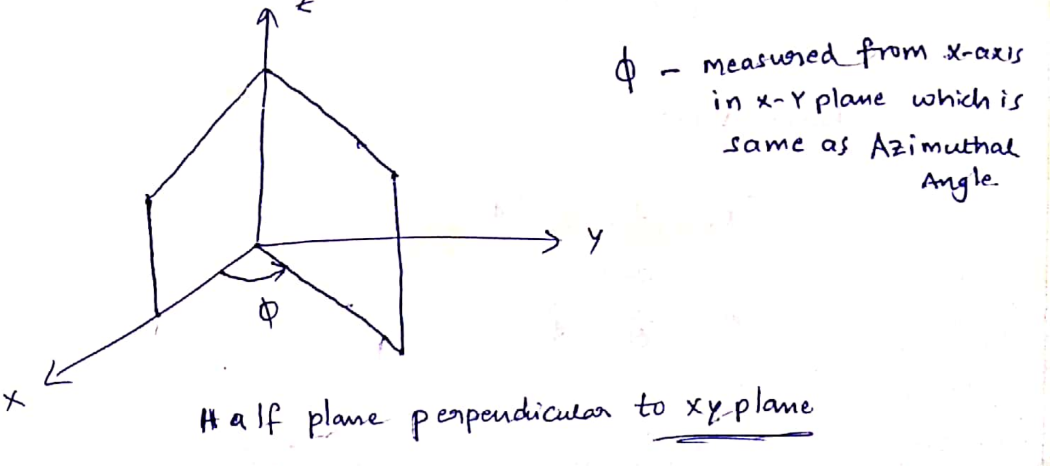
$\phi \rightarrow$ A half plane $\perp$r to XY plane Containing Z-axis making an angle ϕ with the X-Z plane.



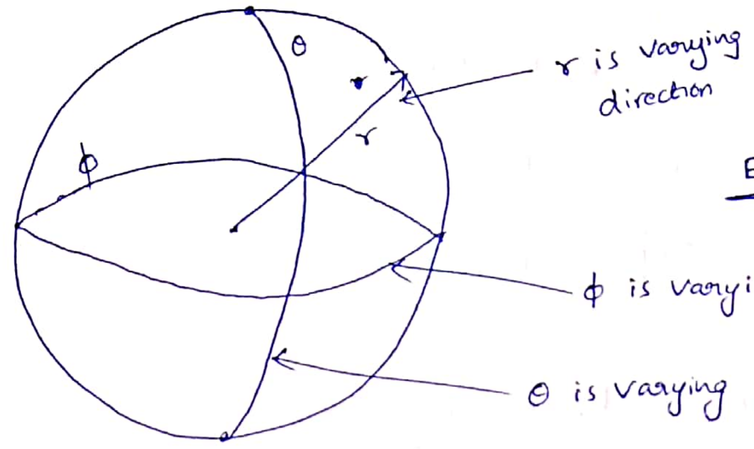
Sphere of radius r with Centre as origin.



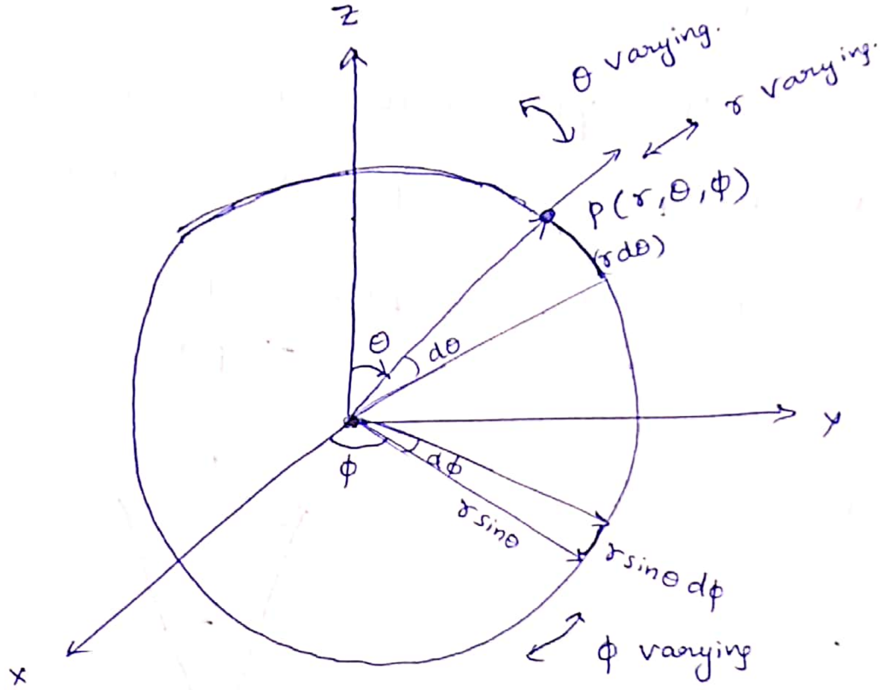
Right Circular Cone with apex at origin.



Half plane perpendicular to xy plane



Ex: Mosambi



Range of Variables.

$$0 \leq r \leq \infty$$

$$0 \leq \phi \leq 2\pi$$

$$0 \leq \theta \leq \pi$$

Base vectors

$$\bar{a}_r , \bar{a}_\theta , \bar{a}_\phi$$

(or)

$$\hat{r} , \hat{\theta} , \hat{\phi}$$

Differential lengths

$dr \rightarrow$ differential length in r direction

$r.d\theta \rightarrow$ differential length in θ direction

$r\sin\theta\, d\phi \rightarrow$ differential length in φ direction

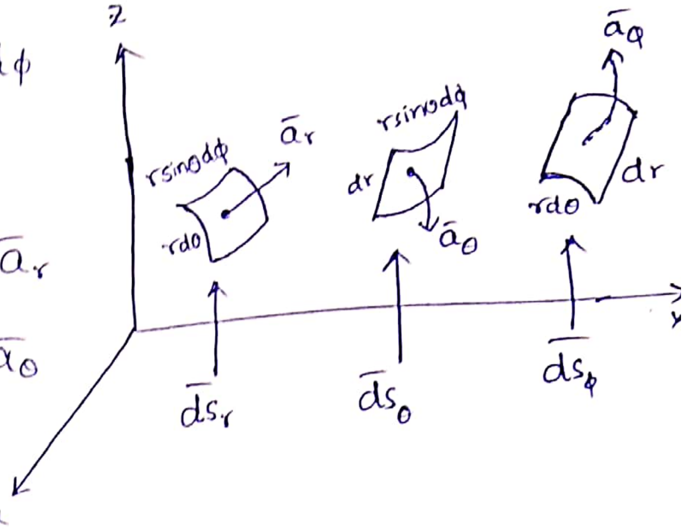
Differential length Vector:

$$\overline{dl} = dr.\bar{a}_r + r\,d\theta.\bar{a}_\theta + r\sin\theta.d\phi\,\bar{a}_\phi$$

$$|\overline{dl}| = \sqrt{(dr)^2 + (r\,d\theta)^2 + (r\sin\theta.d\phi)^2}$$

Differential Volume:

$$dv = r^2\sin\theta\, dr.d\theta.d\phi$$



Differential surface areas:

$$\overline{ds}_r = r^2\sin\theta.d\theta.d\phi.\bar{a}_r$$

$$\overline{ds}_\theta = r\sin\theta.dr.d\phi.\bar{a}_\theta$$

$$\overline{ds}_\phi = r.dr.d\theta.\bar{a}_\phi$$

Cartesian to Spherical:

$$r = \sqrt{x^2+y^2+z^2}$$

$$\theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right]$$

$$\phi = \tan^{-1}(y/x)$$

Spherical to Cartesian

$$x = r\sin\theta\cos\phi$$

$$y = r\sin\theta\sin\phi$$

$$z = r\cos\theta$$

(P) Calculate the volume of the sphere of radius R, using integration.

Sol:

$$\text{Volume}, V = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} dV$$

$$V = \int_0^{2\pi}\int_0^{\pi}\int_0^{R} r^2 \sin\theta \cdot dr \cdot d\theta \cdot d\phi$$

$$= \int_0^{2\pi}\int_0^{\pi} \left[\frac{r^3}{3}\right]_0^R \sin\theta \cdot d\theta \cdot d\phi$$

$$= \frac{R^3}{3}\int_0^{2\pi} \left[-\cos\theta\right]_0^{\pi} d\phi$$

$$= \frac{R^3}{3}\int_0^{2\pi} \left(-\cos\pi + \cos 0\right) d\phi$$

$$= \frac{R^3}{3}\int_0^{2\pi} (1+1) \cdot d\phi$$

$$= \frac{2R^3}{3}\left[\phi\right]_0^{2\pi} = \frac{4\pi R^3}{3}$$

(P) Calculate the surface area of a sphere of radius R by integration.

Sol: Consider the differential surface area normal to r direction.

$$\bar{ds_r} = r^2 \sin\theta \cdot d\theta \cdot d\phi$$

$$S_r = \int_0^{2\pi}\int_0^{\pi} ds_r = \int_0^{2\pi}\int_0^{\pi} r^2 \sin\theta \cdot d\theta \cdot d\phi$$

radius of sphere is constant, $r = R$

$$S_r = \int_0^{2\pi}\int_0^{\pi} R^2 \sin\theta \cdot d\theta \cdot d\phi.$$

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$$S_r = R^2 \int_0^{2\pi}\int_0^{\pi} \sin\theta \cdot d\theta \cdot d\phi$$

$$= R^2 \int_0^{2\pi} (-\cos\theta)_0^{\pi} \, d\phi$$

$$= 2R^2 \left[\phi\right]_0^{2\pi}$$

$$\boxed{S_r = 4\pi R^2.}$$

(P) Convert the point P (3,4,5) from Cartesian to Spherical Co-ordinates.

Sol:

$x = 3$, $y = 4$, $z = 5$

$$r = \sqrt{x^2+y^2+z^2} = \sqrt{9+16+25} = 7.071$$

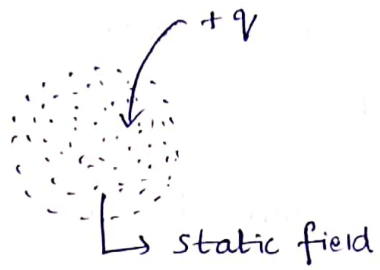
$$\theta = \cos^{-1}\left[\frac{z}{\sqrt{x^2+y^2+z^2}}\right] = \cos^{-1}\left[\frac{5}{\sqrt{50}}\right] = 45^\circ$$

$$\phi = \tan^{-1}\left[\frac{y}{x}\right] = \tan^{-1}\left[\frac{4}{3}\right] = 53.13^\circ$$

$$(r, \theta, \phi) = (7.071, 45^\circ, 53.13^\circ)$$

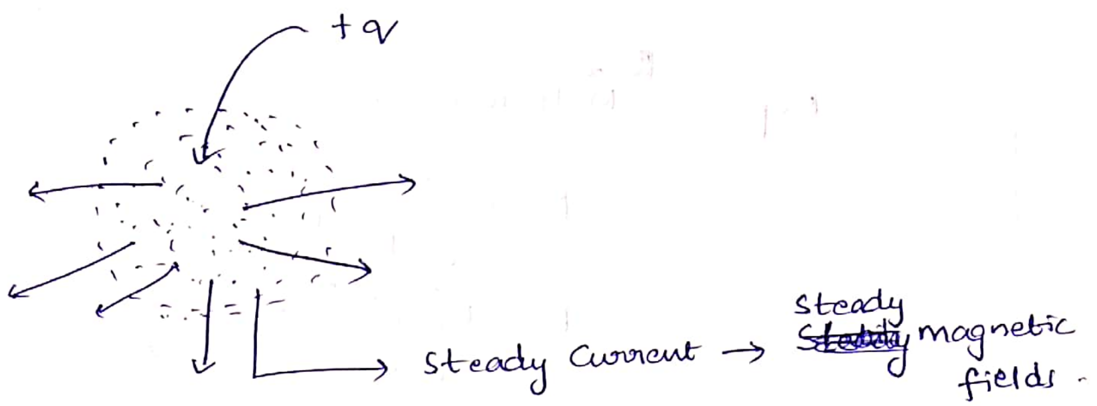
The most fundamental problem in emf is force i.e.,

①



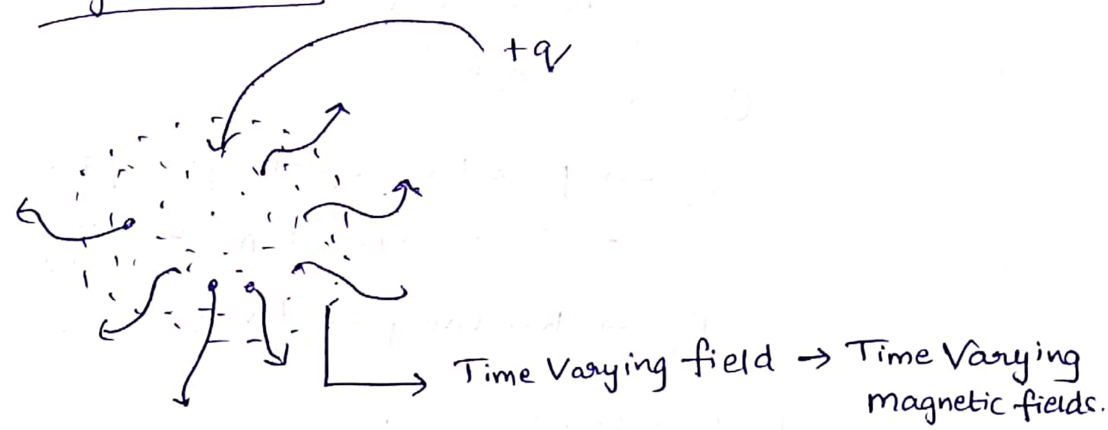
The study of force acting on test charge due to the Collection of Source charges which are at rest is called "Electrostatics".

②



The study of force acting on test charge due to the Collection of Source Charges which are in uniform motion is called "Magneto Statics".

③



→ The study of force acting on test charge due to the Collection of Source Charges which are in acceleration is Called Electrodynamics. or time Varying Magnetic fields.

Coulomb's Law:

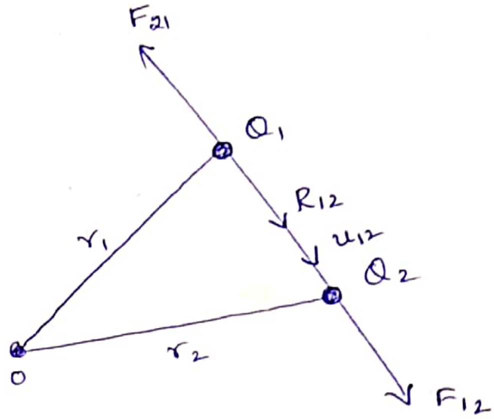
(Calomel charles Coulomb - french army Engineer)

It states that, the force between the two point charges Q_1 and Q_2 is given by

(i) acts along the line joining the two point charges

(ii) directly proportional to the product of the two point charges Q_1 and Q_2

(iii) inversely proportional to the square of the distance between two point charges.



Expressed Mathematically

$$F \propto \left[\frac{Q_1 Q_2}{R^2}\right] u_{12}$$

$$F = K\left[\frac{Q_1 Q_2}{R^2}\right] u_{12}$$

where $K = \frac{1}{4\pi\varepsilon}$

u_{12} - unit vector directed from Q_1 & Q_2.

$\varepsilon \rightarrow$ permitivity of the medium in which charges are located (F/m)

$$\boxed{\varepsilon = \varepsilon_0 \varepsilon_r}$$

$\varepsilon_0 \rightarrow$ permitivity of the free space (or) air (or) vaccume.

$= 8.854 \times 10^{-12}$ F/m $= \frac{10^{-9}}{36\pi}$ F/m

$\varepsilon_r \rightarrow$ Relative permitivity.

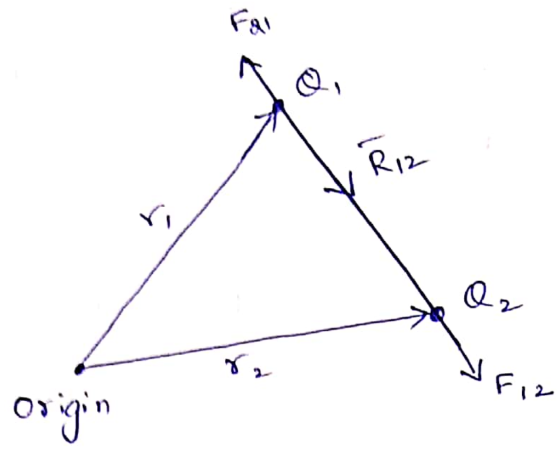
In free space, $\varepsilon_r = 1$

$\varepsilon = \varepsilon_0 \varepsilon_r = \varepsilon_0$.

$$\boxed{\therefore \quad F_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}}\ u_{12}$$

Vector form of Coulomb's Law :

→ If point charges Q_1 and Q_2 are located at points having a position vectors r_1 and r_2, then the force F_{12} on Q_2 due to Q_1



$$\boxed{F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \bar{a}_{R12}} \rightarrow (1)$$

Where $\bar{R}_{12} = r_2 - r_1$

$$R = |R_{12}|$$

$$\bar{a}_{R12} = \frac{\bar{R}_{12}}{|R_{12}|} = \frac{\bar{R}_{12}}{R} \rightarrow (2)$$

(2) in (1)

$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \frac{\bar{R}_{12}}{R}$$

$$= \frac{Q_1 Q_2}{4\pi\epsilon_0 R^3} R_{12}$$

$$\boxed{F_{12} = \frac{Q_1 Q_2 (r_2 - r_1)}{4\pi\epsilon_0 |r_2 - r_1|^3}}$$

The force F_{21} on Q_1 due to Q_2 is given by

$$F_{21} = |F_{12}| \bar{a}_{R21} = |F_{12}| (-\bar{a}_{R12})$$

$$\boxed{F_{21} = -F_{12}}$$

Since $\bar{a}_{R21} = -\bar{a}_{R12}$

(P) Let us illustrate the use of the vector form of Coulomb's law by locating a charge of $Q_1 = 3\times10^{-4}C$ at $M(1,2,3)$ and a charge of $Q_2 = -10^{-4}C$ at $N(2,0,5)$ in a vaccum. We desire the force exeerted on Q_2 by Q_1.

Sol:

$$R_{12} = r_2 - r_1 = (2-1)\bar{a}_x + (0-2)\bar{a}_y + (5-3)\bar{a}_z$$

$$= a_{\bar{x}} - 2\bar{a}_y + 2\bar{a}_z$$

$$|R_{12}| = \sqrt{1^2 + (-2)^2 + (2)^2} = \sqrt{9} = 3$$

$$\bar{a}_{R_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}$$

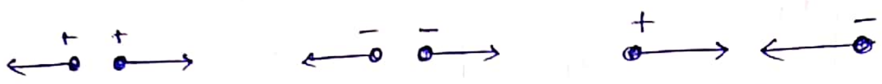
$$F_{12} = \frac{Q_1 Q_2}{4\pi\epsilon_0 R^2} \cdot \frac{R_{12}}{|R_{12}|}$$

$$= \frac{3\times10^{-4}(-10^{-4})}{4\pi\left(\frac{1}{36\pi}\right)10^{-9}\times3^2}\left(\frac{\bar{a}_x - 2\bar{a}_y + 2\bar{a}_z}{3}\right)$$

$$\boxed{F_{12} = -10\bar{a}_x + 20\bar{a}_y - 20\bar{a}_z}$$

NOTE

→



(a) (b) (c)

Fig (a),(b) Like charges repel
(c) Unlike charges attract.

→ If we have more than two point charges, we can use the principle of superposition to determine the force on a particular charge.

→ The resultant force F on a charge located at a point with its position vector r.

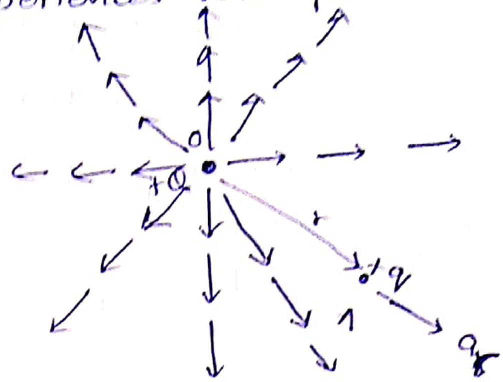
$$F = \frac{QQ_1(r-r_1)}{4\pi\epsilon_0|r-r_1|^3} + \frac{QQ_2(r-r_2)}{4\pi\epsilon_0|r-r_2|^3} + \cdots + \frac{QQ_N(r-r_N)}{4\pi\epsilon_0|r-r_N|^3}.$$

N - Charges
r_N - position vectors.

$$F = \frac{Q}{4\pi\epsilon_0}\sum_{K=1}^{N}\frac{Q_K(r-r_K)}{|r-r_K|^3}$$

Electric Field Intensity:

"The electric field intensity (or electric field strength) E is the force that a unit positive charge experiences when placed in an electric field.



Thus

$$E = \lim_{Q \to 0} \frac{F}{Q}$$

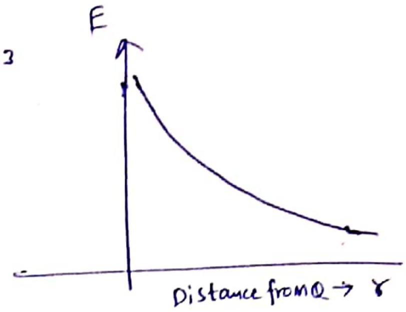
or simply $$\boxed{E = \frac{F}{Q}}$$

For $Q > 0$, the electric field intensity E is obviously in the direction of the force F and is measured in newtons/coulomb (or) Volts/meter.

→ The electric field intensity at point r due to a point charge located at r'

$$E = \frac{Q}{4\pi\varepsilon_0 R^2} a_R = \frac{Q(r - r')}{4\pi\varepsilon_0 |r - r'|^3}$$

or simply $$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} \bar{a}_r}$$



→ For N point charges $Q_1, Q_2 \ldots Q_N$ located at $r_1, r_2 \ldots r_N$, The EFI at point r is obtained

$$E = \frac{Q_1(r - r_1)}{4\pi\varepsilon_0 |r - r_1|^3} + \frac{Q_2(r - r_2)}{4\pi\varepsilon_0 |r - r_2|^3} + \ldots + \frac{Q_N(r - r_N)}{4\pi\varepsilon_0 |r - r_N|^3}$$

(or)

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \sum_{K=1}^{N} \frac{Q_K(r - r_K)}{|r - r_K|^3}}$$

Electric Field Intensity:

(P) point charges 1mC and -2mC are located at (3,2,-1) and (-1,-1,4), respectively. Calculate the electric force on a 10nC Charge located at (0,3,1) and the electric field intensity at that point.

Sol:

$$F = \sum_{K=1,2} \frac{Q Q_K}{4\pi\varepsilon_0 R^2} a_R = \sum_{K=1,2} \frac{Q Q_K (r - r_K)}{4\pi\varepsilon_0 |r - r_K|^3}$$

$$= \frac{Q}{4\pi\varepsilon_0} \left\{ \frac{Q_1 (r - r_1)}{|r - r_1|^3} + \frac{Q_2 (r - r_2)}{|r - r_2|^3} \right\}$$

$$= \frac{Q}{4\pi\varepsilon_0} \left\{ \frac{10^{-3} [(0,3,1) - (3,2,-1)]}{|(0,3,1) - (3,2,-1)|^3} - \frac{2\times 10^{-3} [(0,3,1) - (-1,-1,4)]}{|(0,3,1) - (-1,-1,4)|^3} \right\}$$

$$= \frac{10\times 10^{-9}}{4\pi \times \frac{1}{36\pi \times 10^9}} \left\{ \frac{10^{-3}(-3,1,2)}{|(-3,1,2)|^3} - \frac{2\times 10^{-3}(1,4,-3)}{|(1,4,-3)|^3} \right\}$$

$$\boxed{F = -6.521\,\bar{a}_x - 3.71\,\bar{a}_y + 7.509\,\bar{a}_z \text{ mN.}}$$

At that point,

$$E = \frac{F}{Q} = \frac{-6.521\,\bar{a}_x - 3.71\,\bar{a}_y + 7.509\,\bar{a}_z \text{ mN.}}{10\times 10^{-9}}$$

$$\boxed{E = -651.2\,\bar{a}_x - 371.3\,\bar{a}_y + 750.9\,a_z \text{ KV/m}}$$

Continuous distribution of charge:

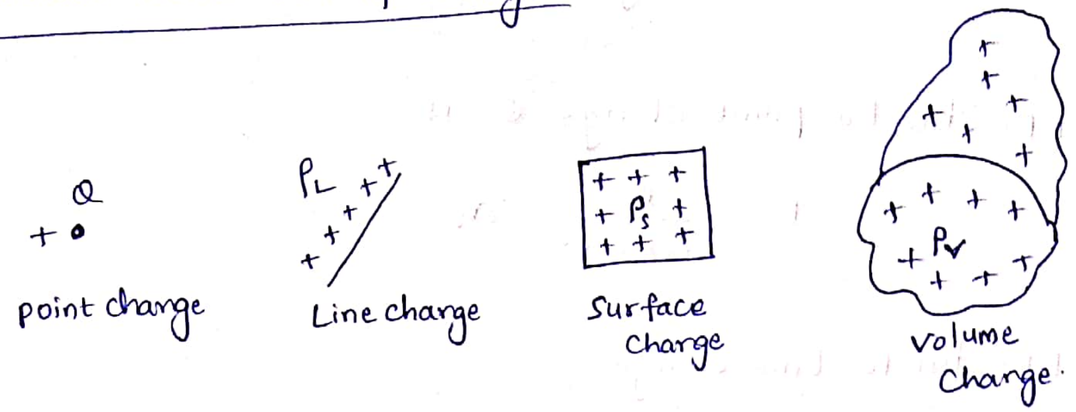


Fig: Various charge distributions.

→ If the charge is distributed along a line with line charge density 'ρ_L' C/m, it is called line charge distribution.

$$\boxed{\rho_L = \frac{dQ}{dL} \Rightarrow dQ = \rho_L dL \Rightarrow Q = \int_L \rho_L \cdot dl}$$ (line charge)

→ If the charge is continuously distributed over a surface with surface charge density 'ρ_s' C/m², it is called surface charge distribution.

$$\boxed{\rho_s = \frac{dQ}{ds} \Rightarrow dQ = \rho_s \cdot ds \Rightarrow Q = \int_S \rho_s \cdot ds}$$ (surface charge)

→ If the charge is distributed over a volume with volume charge density 'ρ_v' C/m³, it is called volume charge distribution.

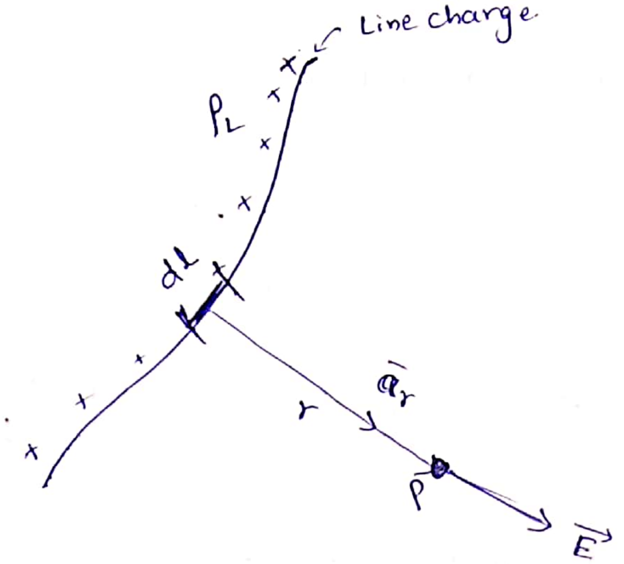
$$\boxed{\rho_v = \frac{dQ}{dv} \Rightarrow dQ = \rho_v \, dv \Rightarrow Q = \int_V \rho_v \cdot dv}$$ (volume charge)

EFI due to Various Charge Distributions:

EFI due to point charge 'Q' is

$$\bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\bar{a}_r .$$

(1) EFI due to Line charge:



→ The charge dQ on the differential length dl is

$$dQ = \rho_L \cdot dL$$

→ The differential electric field $d\bar{E}$ at point 'p' due to 'dQ' is

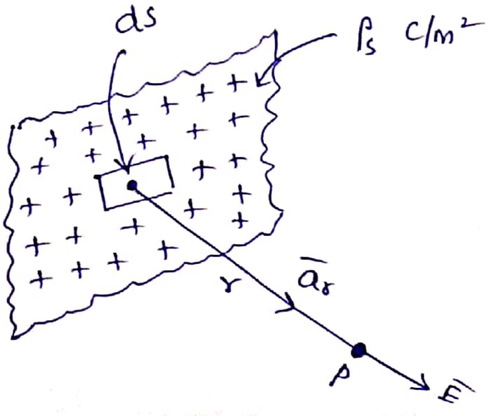
$$dE = \frac{dQ}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

$$dE = \frac{\rho_L\, dL}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

→ Hence, the total $\bar{E}$ at a point 'p' due to line charge is

$$\boxed{\bar{E} = \int_L \frac{\rho_L\, dL}{4\pi\epsilon_0 r^2}\,\bar{a}_r}$$

(2) EFI due to Surface charge:



→ The charge dQ is

$$dQ = \rho_s \cdot ds$$

→ The differential electric field dE at point 'p' due to 'dQ' is

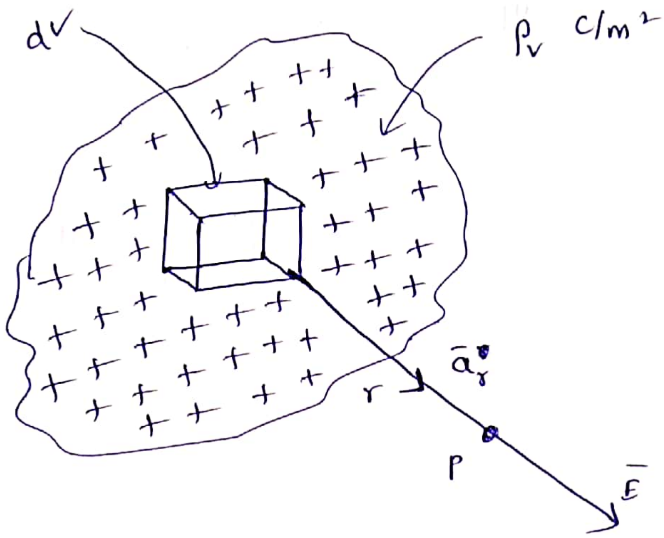
$$dE = \frac{dQ}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

$$dE = \frac{\rho_s\, ds}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

The total Electrical field intensity $\bar{E}$ at a point 'p' is

$$\boxed{\vec{E} = \int_s \frac{\rho_s \cdot ds}{4\pi\epsilon_0 r^2}\,\bar{a}_r}$$

3. EFI due to Volume charge:



→ The charge dQ is

$$dQ = \rho_v \cdot dV$$

→ The differential electric field $\bar{dE}$ at a point 'p' due to 'dQ' is

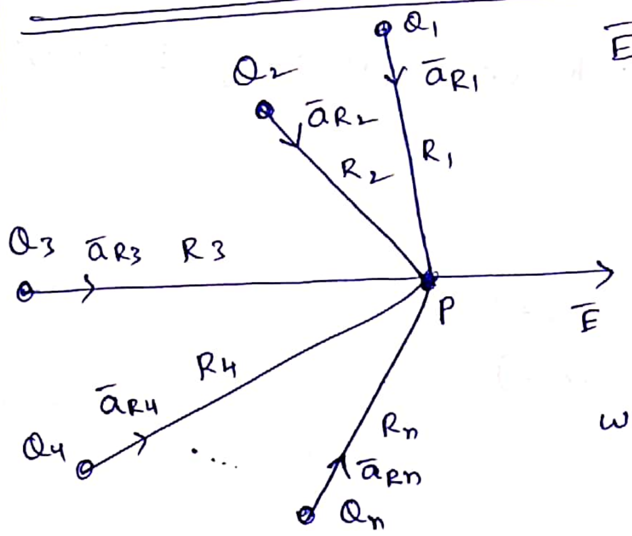
$$\bar{dE} = \frac{dQ}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

$$\bar{dE} = \frac{\rho_v\, dV}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

→ The total electric field intensity $\bar{E}$ at a point 'p' is

$$\boxed{\bar{E} = \int_V \frac{\rho_v \cdot dV}{4\pi\epsilon_0 r^2}\,\bar{a}_r}$$

Electric field due to discrete charge:



$\bar{E}$ due to no. of charges

$$\bar{E} = \bar{E}_1 + \bar{E}_2 + \dots + E_n$$

$$= \frac{Q_1}{4\pi\epsilon_0 R_1^2}\bar{a}_{R1} + \frac{Q_2}{4\pi\epsilon_0 R_2^2}\bar{a}_{R2} + \dots + \frac{Q_n}{4\pi\epsilon_0 R_n^2}\bar{a}_{Rn}$$

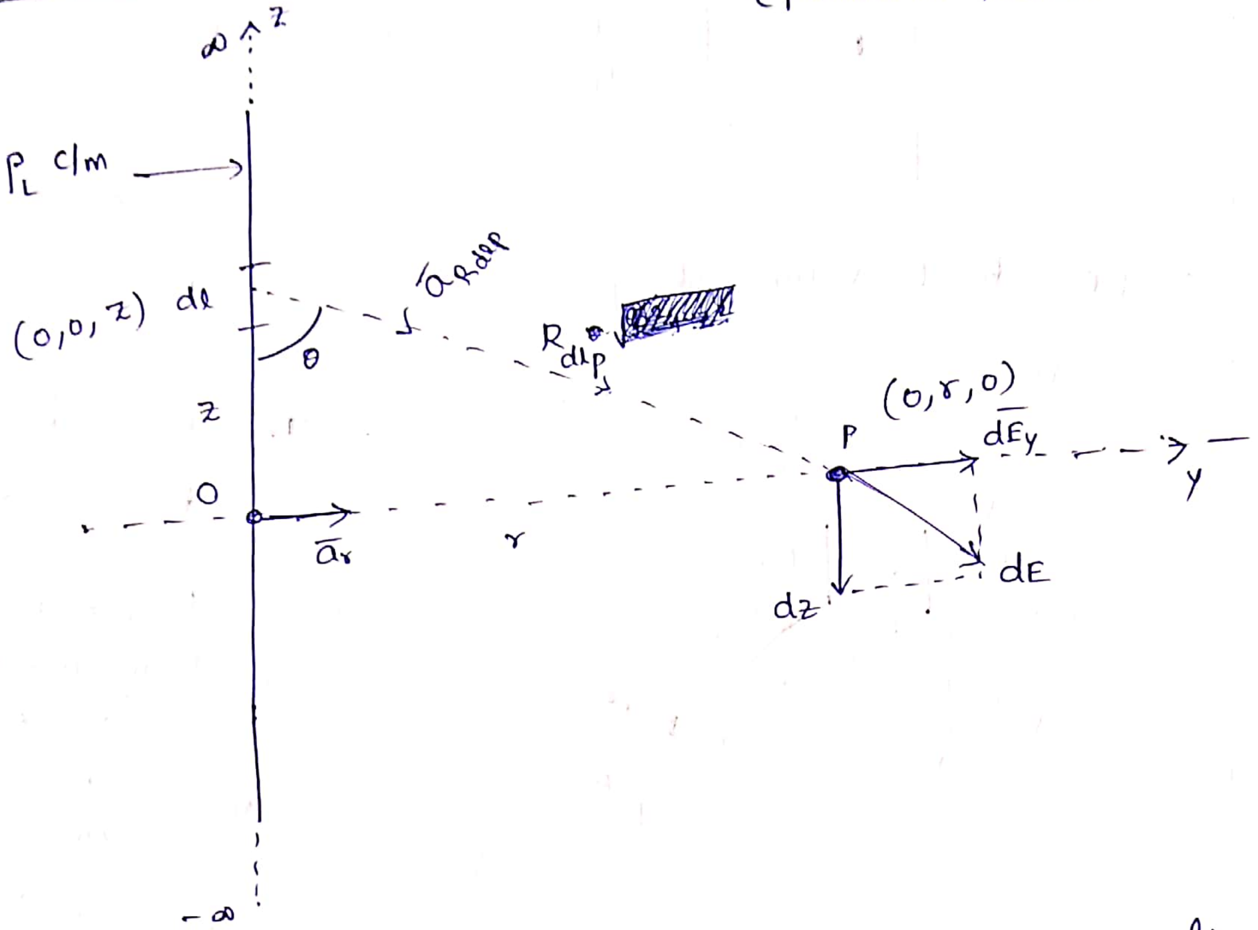
$$\boxed{\bar{E} = \frac{1}{4\pi\epsilon_0}\sum_{L=1}^{n} \frac{Q_L}{R_L^2}\,\bar{a}_{RL}}$$

where Q_L – L^{th} charge

R_L – distance of L^{th} charge to p

$\bar{a}_{RL}$ – unit Vector directed from L^{th} charge to p.

Electric Field Intensity due to infinite line charge :-
(placed on z-axis).



→ Consider an infinitely long straight line carrying uniform line charge having density ρ_L C/m. which is placed along z-axis from $-\infty$ to $+\infty$. Hence this is called infinite line charge.

→ Let the point P is on y-axis at which $\bar{E}$ is to be determined.

→ Distance of point P from the origin is 'r'.

→ Consider a small differential length 'dl' carrying a charge 'dQ' along the line.

$\therefore$ $dl = dz$ [$\because$ The charge is along z-axis]

→ $dQ = \rho_L \cdot dl = \rho_L \cdot dz$.

→ Coordinates of dQ is $(0,0,z)$

→ Coordinates of P is $(0,r,0)$

Distance Vector :

$$\bar{R}_{dlp} = \bar{R}_p - \bar{R}_{dl} = r\,\bar{a}_y - z\,\bar{a}_z$$

Magnitude of distance vector

$$|\bar{R}_{dlp}| = \sqrt{r^2 + z^2}$$

Unit Vector is

$$\bar{a}_{R_{dlp}} = \frac{\bar{R}_{dlp}}{|\bar{R}_{dlp}|} = \frac{r\,\bar{a}_y - z\,\bar{a}_z}{\sqrt{r^2 + z^2}}$$

The different electric field dueto 'dQ' is

$$\bar{dE} = \frac{dQ}{4\pi\varepsilon_0 R_{dlp}^2} \cdot \bar{a}_{R_{dlp}}$$

$$\bar{dE} = \frac{\rho_L \cdot dz}{4\pi\varepsilon_0 \left(\sqrt{r^2+z^2}\right)^2} \left[\frac{r\,\bar{a}_y - z\,\bar{a}_z}{\sqrt{r^2+z^2}}\right]$$

The total electric field $\bar{E}$ is

$$\bar{E} = \int_{-\infty}^{+\infty} \frac{\rho_L}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \cdot r \cdot dz \cdot \bar{a}_y$$

[$\because$ There will be no z Component of $\bar{E}$ at 'p'.]

$z = r\tan\theta \Rightarrow$ | $r = \frac{z}{\tan\theta}$

~~$r = \frac{z}{\tan\theta}$~~

$dz = r\sec^2\theta \cdot d\theta$

$\tan\theta = \frac{z}{r}$

$\theta = \tan^{-1}\frac{z}{r}$

Scanned by CamScanner

For $z = -\infty$; $\theta = \tan^{-1}(-\infty) = -\pi/2$

For $z = \infty$; $\theta = \tan^{-1}(\infty) = +\pi/2$.

$$\bar{E} = \int_{\theta=-\pi/2}^{\theta=\pi/2} \frac{\rho_L}{4\pi\epsilon_0 [r^2 + r^2\tan^2\theta]^{3/2}} \cdot r \cdot (r \sec^2\theta \, d\theta)\, \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{r^2 \sec^2\theta \cdot d\theta}{(r^{\cancel{2}})^{3/\cancel{2}} [1+\tan^2\theta]^{3/2}} \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\epsilon_0} \int_{-\pi/2}^{\pi/2} \frac{\cancel{r^2} \sec^2\theta \cdot d\theta}{r^{\cancel{3}\,1} (\sec^{\cancel{2}}\theta)^{3/\cancel{2}}} \cdot \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\epsilon_0 r} \int_{-\pi/2}^{\pi/2} \frac{1}{\sec\theta} \cdot d\theta \cdot \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\epsilon_0 r} \int_{-\pi/2}^{\pi/2} [\cos\theta] \, d\theta \cdot \bar{a}_y$$

$$= \frac{\rho_L}{4\pi\epsilon_0 r} [\sin\theta]_{-\pi/2}^{\pi/2} \cdot \bar{a}_y$$

$$= \frac{\rho_L \bar{a}_y}{4\pi\epsilon_0 r} \left[\sin\pi/2 - \sin(-\pi/2) \right]$$

$$= \frac{\rho_L \bar{a}_y}{4\pi\epsilon_0 r} [1 - (-1)]$$

$$= \frac{2\rho_L \bar{a}_y}{2\,4\pi\epsilon_0 r}$$

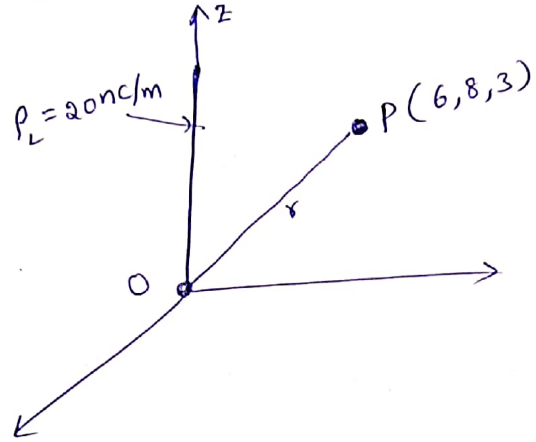
$$\boxed{\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 r} \bar{a}_y} \quad V/m$$

$\bar{a}_y = \bar{a}_r$ [$\because$ $\bar{a}_y$ is the unit vector along the distance vector r].

so, $$\boxed{\bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\,\bar{a}_r \ \ V/m}$$

(P) A uniform line charge, infinite is extent with $\rho_L = 20\,nC/m$ along z-axis. Find $\bar{E}$ at $(6,8,3)$ m.

Sol:



→ $\bar{E}$ cannot have any component along z direction as line charge is along z-axis. So, we do not consider z-coordinate while calculating $\bar{r}$.

$$\bar{r}_{op} = 6\bar{a}_x + 8\bar{a}_y$$

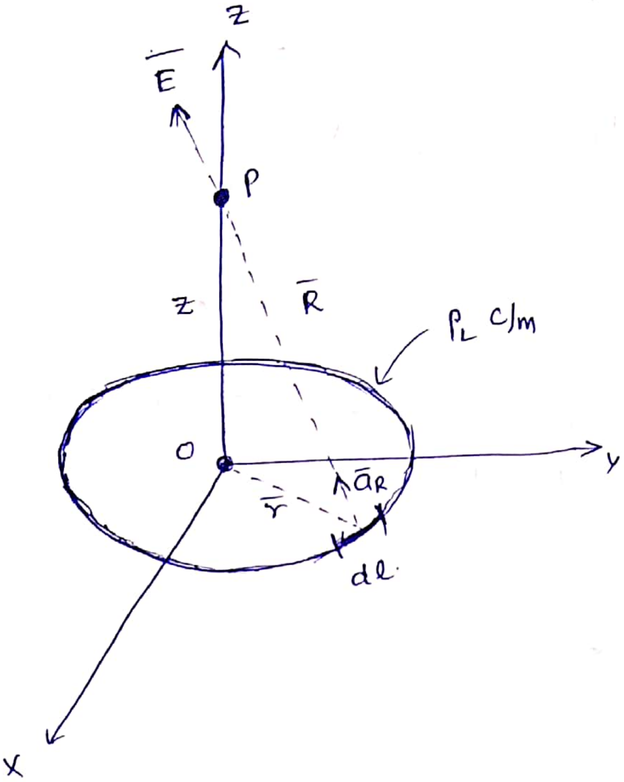
$$|\bar{r}_{op}| = \sqrt{6^2 + 8^2} = 10$$

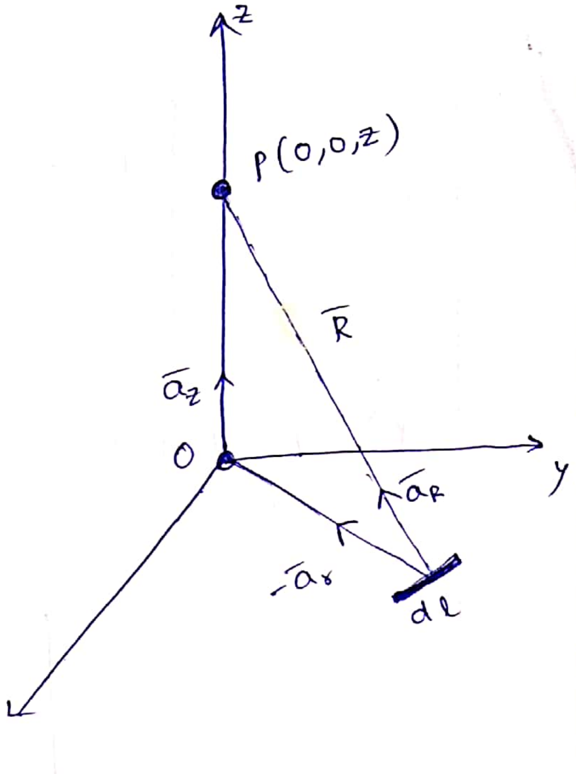
$$\bar{a}_r = \frac{6\bar{a}_x + 8\bar{a}_y}{10} = 0.6\,\bar{a}_x + 0.8\,\bar{a}_y$$

$$\text{so, } \bar{E} = \frac{\rho_L}{2\pi\varepsilon_0 r}\bar{a}_r = \frac{20\times10^{-9}}{2\pi\times8.854\times10^{-12}\times10}[0.6\bar{a}_x + 0.8\bar{a}_y]$$

$$\boxed{\bar{E} = 21.57\,\bar{a}_x + 28.76\,\bar{a}_y} \ \ V/m$$

Electric Field Intensity due to Charged Circular Ring :





→ Consider a charged circular ring of radius 'r' placed in xy plane with centre at origin, carrying a charge uniformly along its circumference.

→ Charge density is ρ_L C/m.

→ point 'p' at a ⊥r distance 'z' from the ring.

→ Consider a small differential length dl on this ring.

$$dQ = \rho_L \, dl$$

$$\bar{dE} = \frac{\rho_L \, dl}{4\pi\varepsilon_0 R^2} \cdot \bar{a}_R$$

$$\bar{dE} = \frac{\rho_L \cdot (r\,d\phi)}{4\pi\varepsilon_0 R^2} \bar{a}_R$$

[Consider Cylindrical Coordinate system. so, $dl = r\,d\phi$]

Now $R^2 = r^2 + z^2$

$$\bar{R} = -r\bar{a}_r + z \cdot \bar{a}_z$$

$$\bar{dE} = \frac{\rho_L \cdot r\, d\phi}{4\pi\varepsilon_0 (r^2+z^2)} \cdot \bar{a}_R$$

$$\left[\because \bar{a}_R = \frac{\bar{R}}{|R|}\right]$$

$$= \frac{\rho_L\, r\, d\phi}{4\pi\varepsilon_0 (r^2+z^2)} \frac{\bar{R}}{|R|}$$

$$= \frac{\rho_L\, r\, d\phi}{4\pi\varepsilon_0 (r^2+z^2)} \left[\frac{-r\cdot\bar{a}_r + z\cdot\bar{a}_z}{\sqrt{r^2+z^2}}\right]$$

$$\bar{dE} = \frac{\rho_L\, r\cdot d\phi}{4\pi\varepsilon_0} \frac{(-r\cdot\bar{a}_r + z\cdot\bar{a}_z)}{(r^2+z^2)^{3/2}}$$

neglect $\bar{a}_r$ component

$$\bar{dE} = \frac{\rho_L\, r\, d\phi}{4\pi\varepsilon_0} \frac{z\cdot\bar{a}_z}{(r^2+z^2)^{3/2}}$$

$$\bar{E} = \int_{\phi=0}^{2\pi} \frac{\rho_L\, r\cdot d\phi}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} z\cdot a_z$$

$$= \frac{\rho_L\cdot r\cdot z}{4\pi\varepsilon_0 (r^2+z^2)^{3/2}} \left[\phi\right]_0^{2\pi} \bar{a}_z$$

$$= \frac{\rho_L\cdot r\cdot z}{\cancel{4}2\cancel{\pi}\varepsilon_0 (r^2+z^2)^{3/2}} [\cancel{2\pi}]\, \bar{a}_z$$

$$\boxed{\therefore \bar{E} = \frac{\rho_L\, r\, z}{2\varepsilon_0 (r^2+z^2)^{3/2}} \cdot \bar{a}_z}\ \text{V/m}.$$

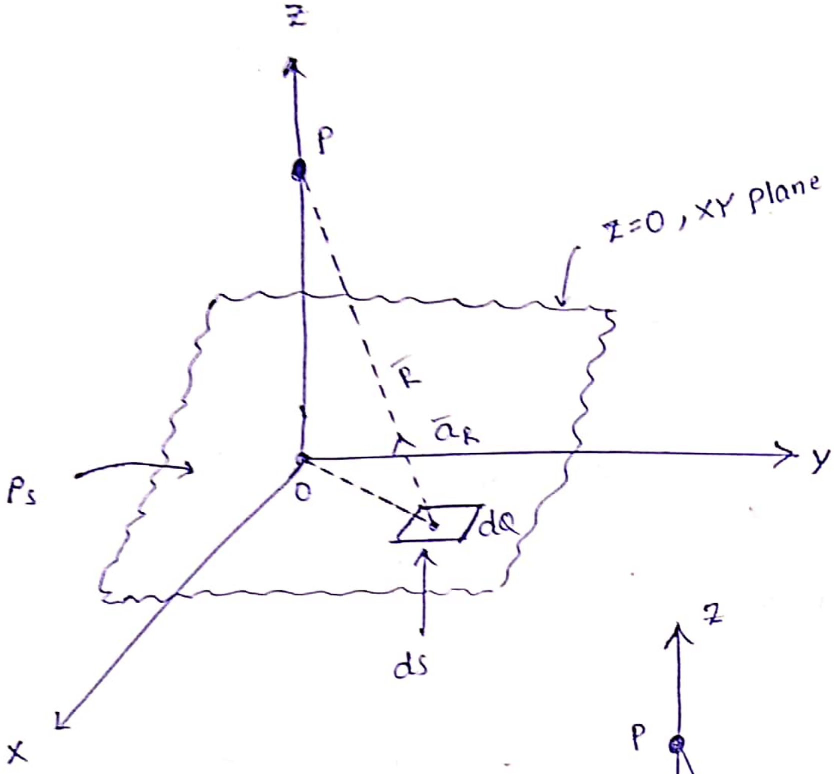
NOTE At Centre of the Circle i.e $z=0$

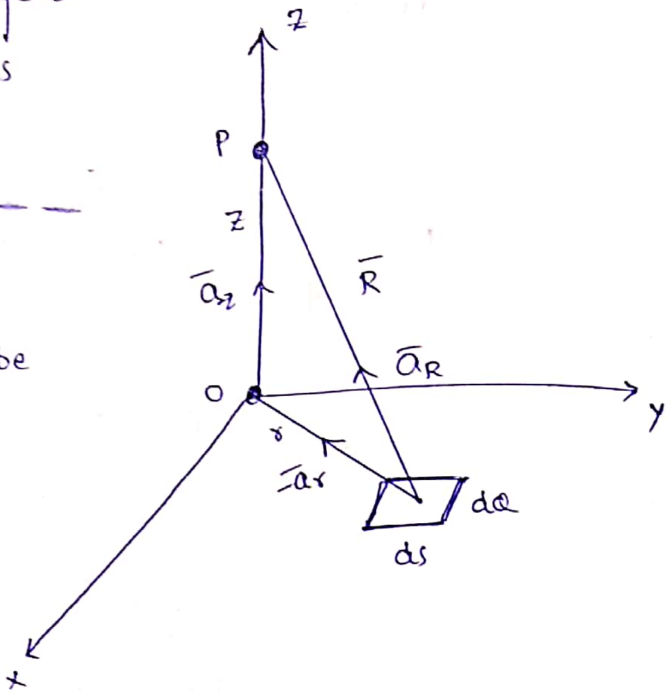
$$\boxed{\bar{E} = 0}$$

Electric field Intensity due to infinite Sheet of Charge:

Consider an infinite sheet of charge having uniform charge density ρ_s C/m² placed in XY-plane

$$d\vec{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\,\bar{a}_R$$





~~$dQ = \rho_s \cdot ds$~~

The point 'p' at which $\bar{E}$ can be calculated is on Z-axis.

$$dQ = \rho_s \cdot ds$$
$$= \rho_s \cdot r \cdot dr\, d\phi$$

$$\bar{R} = -r\,\bar{a}_r + z\,\bar{a}_z$$

$$|\bar{R}| = \sqrt{r^2+z^2} \quad ; \quad R^2 = r^2+z^2$$

$$\bar{a}_R = \frac{\bar{R}}{|\bar{R}|} = \frac{-r\cdot\bar{a}_r + z\cdot\bar{a}_z}{\sqrt{r^2+z^2}}$$

Hence, $d\bar{E} = \dfrac{dQ}{4\pi\varepsilon_0 R^2}\,\bar{a}_R$

$$= \frac{\rho_s \cdot r \cdot dr \cdot d\phi}{4\pi\varepsilon_0 (r^2+z^2)}\left(\frac{-r\cdot\bar{a}_r + z\cdot\bar{a}_z}{\sqrt{r^2+z^2}}\right)$$

$$\therefore\ \bar{E} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty} d\bar{E}$$

All $\bar{a}_r$ components of $\bar{E}$ are going to cancel each other.

$$\bar{E} = \int_{\phi=0}^{2\pi}\int_{r=0}^{\infty} \frac{\rho_s\cdot r\cdot dr\cdot d\phi}{4\pi\varepsilon_0\ (r^2+z^2)^{3/2}}\cdot z\cdot\bar{a}_z$$

put $\boxed{u^2 = r^2+z^2}$ ~~hence~~

$2u\,du = 2r\cdot dr$

$\boxed{u\,du = r\cdot dr}$

For $r=0,\ u=z$

$r=\infty,\ u=\infty$

$$\bar{E} = \int_{0}^{2\pi}\int_{u=z}^{\infty} \frac{\rho_s}{4\pi\varepsilon_0}\cdot\frac{u\cdot du\ d\phi}{(u^2)^{3/2}}\cdot z\cdot\bar{a}_z$$

$$= \int_{0}^{2\pi}\int_{u=z}^{\infty} \frac{\rho_s}{4\pi\varepsilon_0}\ \frac{\cancel{u}\cdot du\cdot d\phi}{u^{\cancel{3}2}}\ z\cdot\bar{a}_z$$

$$= \frac{\rho_s\ z}{4\pi\varepsilon_0}\cdot\int_{0}^{2\pi}\int_{u=z}^{\infty} u^{-2}\cdot du\cdot d\phi\cdot\ \bar{a}_z$$

$$\boxed{\int u^{-2}du = \frac{u^{-2+1}}{-2+1} = \frac{u^{-1}}{-1} = -\frac{1}{u}}$$

$$= \frac{\rho_s\ z}{4\pi\varepsilon_0}\int_{0}^{2\pi}\left[\frac{-1}{u}\right]_{z}^{\infty}\cdot d\phi\cdot\bar{a}_z$$

$$= \frac{\rho_s\ z}{4\pi\varepsilon_0}\int_{0}^{2\pi}\left[\frac{-1}{\infty}+\frac{1}{z}\right]\cdot d\phi\cdot\bar{a}_z$$

$$\bar{E} = \frac{\rho_s \cancel{z}}{4\pi\epsilon_0 \cancel{z}} [\phi]_0^{2\pi} \bar{a}_z$$

$$= \frac{\rho_s (2\cancel{\pi})}{2\cancel{4}\cancel{\pi}\epsilon_0} \bar{a}_z$$

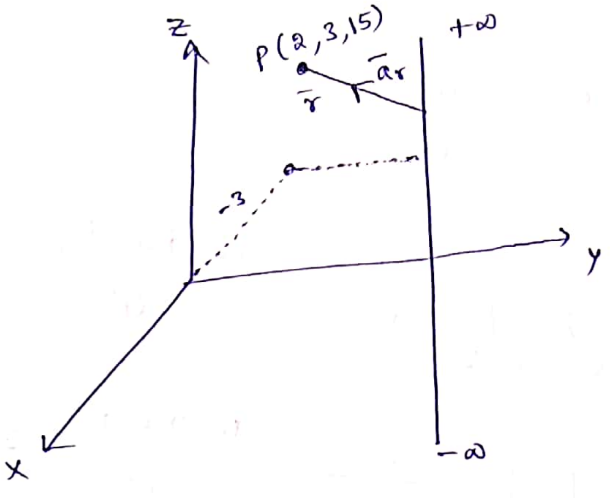
$$\boxed{\bar{E} = \frac{\rho_s}{2\epsilon_0} \bar{a}_z} \quad V/m \quad \text{for points above XY plane}$$

$$\boxed{\bar{E} = \frac{-\rho_s}{2\epsilon_0} \bar{a}_z} \quad V/m \quad \text{for points below XY plane.}$$

problems

(P1) A uniform line charge, $\rho_L = 25$ nC/m lies on the line $x = -3$m and $y = 4$m in free space. Find $\bar{E}$ at a point $(2,3,15)$ m.

Sol:



EFI due to infinite line charge

$$\bar{E} = \frac{\rho_L}{2\pi\epsilon_0 x} \bar{a}_r$$

$$\bar{r} = [2-(-3)]\bar{a}_x + [3-4]\bar{a}_y$$

$$= 5\bar{a}_x - \bar{a}_y$$

$$|\bar{r}| = \sqrt{5^2 + (-1)^2} = \sqrt{25+1} = \sqrt{26}$$

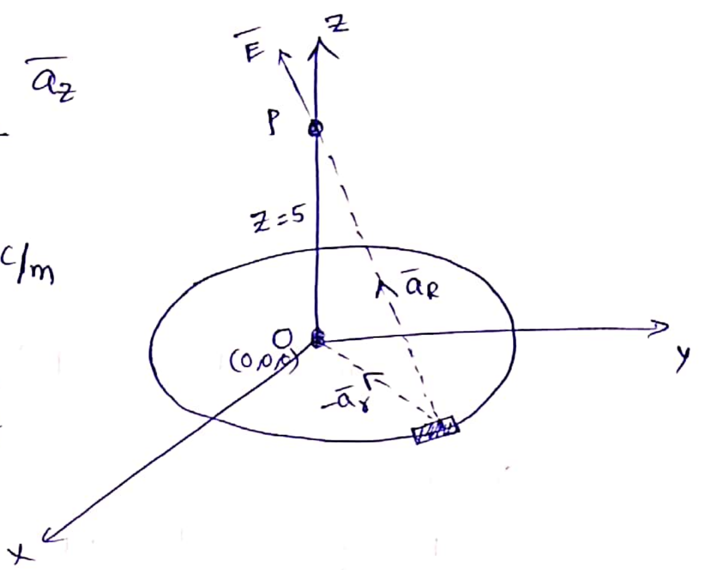
$$\bar{a}_r = \frac{\bar{r}}{|\bar{r}|} = \frac{5\bar{a}_x - \bar{a}_y}{\sqrt{26}} = 0.98\,\bar{a}_x - 0.196\,\bar{a}_y$$

$$\bar{E} = \frac{25 \times 10^{-9}}{2\pi \times 8.854 \times 10^{-12} \times \sqrt{26}} (0.98\,\bar{a}_x - 0.196\,\bar{a}_y)$$

$$= 88.132\,(0.98\,\bar{a}_x - 0.196\,\bar{a}_y)$$

$$\bar{E} = 86.42\,\bar{a}_x - 17.284\,\bar{a}_y \ \text{V/m}$$

(P2) A circular ring of charge with radius 5m lies in z=0 plane with centre as origin. If the $\rho_L = 10\,nC/m$, then find the point charge Q_x placed at the origin which will produce same $\bar{E}$ at the point (0,0,5)m.



Sol

$$\bar{E} = \frac{\rho_L\, r\, z}{2\epsilon_0 (r^2+z^2)^{3/2}}\,\bar{a}_z$$

$z=5,\ r=5m,\ \rho_L = 10\,nC/m$

$$\bar{E} = \frac{10 \times 10^{-9} \times 5 \times 5}{2(8.854 \times 10^{-12})(5^2+5^2)^{3/2}} \cdot \bar{a}_z$$

$$\bar{E} = 39.93\,\bar{a}_z \ \text{V/m}.$$

P (0,0,5) & Origin (0,0,0)

$$\bar{R}_{op} = R_p - R_o = 5\,\bar{a}_z$$

$$\bar{a}_R = \frac{\bar{R}_{op}}{|R_{op}|} = \frac{5\,\bar{a}_z}{\sqrt{(5)^2}} = \bar{a}_z$$

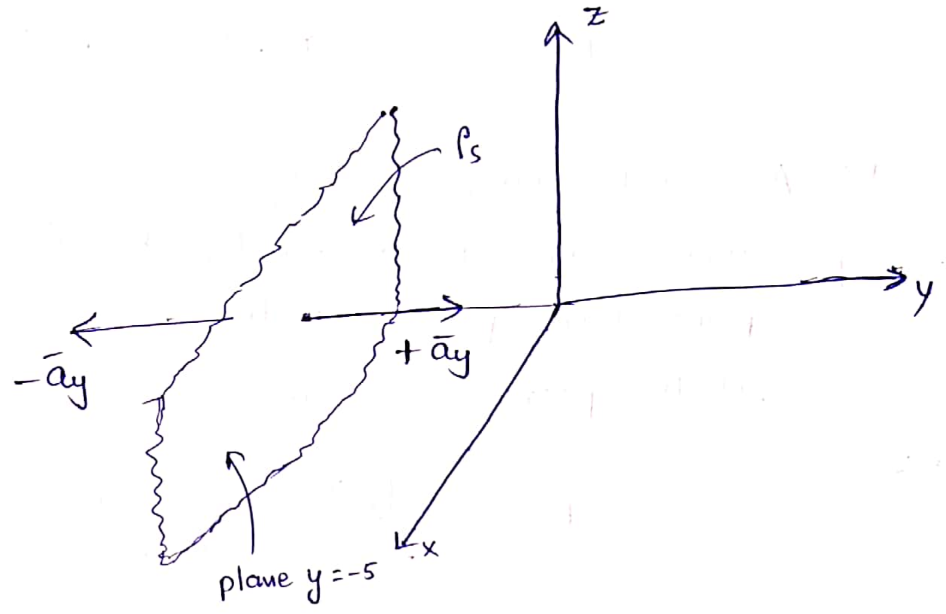
$$\bar{E} \text{ at p due to } Q = \frac{Q}{4\pi\epsilon_0 R^2}\,\bar{a}_R$$

$$39.93\,\bar{a}_z = \frac{Q}{4\pi \times 8.854 \times 10^{-12} \times 5^2} \cdot \bar{a}_z$$

$$\boxed{\therefore Q = 111.071 \ nC}$$

(P3) Charge lies in $y = -5m$ plane in the form of an infinite Square Sheet with a uniform Charge density of $\rho_s = 20 nC/m^2$. Determine $\bar{E}$ at all the points.

Sol.



Case (1):

$y = -5m$ Constant is parallel to xz plane.

Case (2):

For $y > -5$, $\bar{E}$ Component will be along $+\bar{a}_y$ direction.

$$\bar{E} = \frac{\rho_s}{2\varepsilon_0}\bar{a}_r = \frac{\rho_s}{2\varepsilon_0}\bar{a}_y = \frac{20\times10^{-19}}{2\times8.854\times10^{-12}}\bar{a}_y$$

$$\bar{E} = 1129.43\ \bar{a}_y\ V/m.$$

Case (3):

For $y < -5$, $\bar{E}$ component will be along $-\bar{a}_y$ direction.

$$\bar{E} = \frac{\rho_s}{2\varepsilon_0}(-\bar{a}_y) = -1129.43\ \bar{a}_y\ V/m.$$

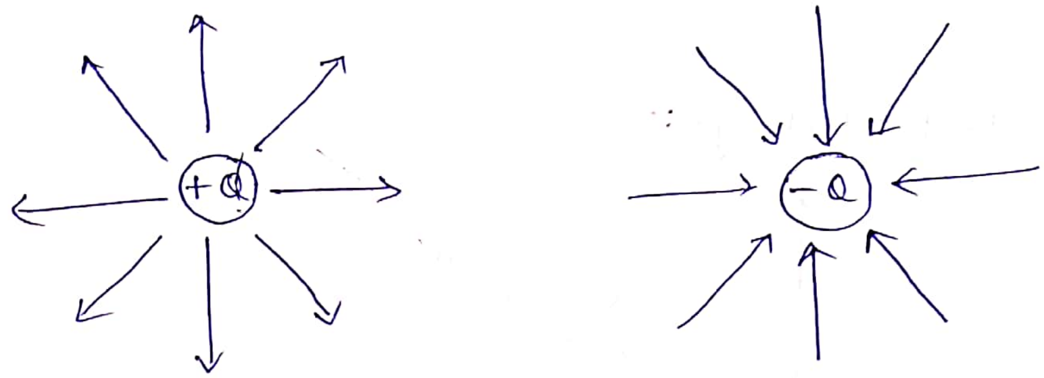
$$|\bar{E}| = \text{constant}.$$

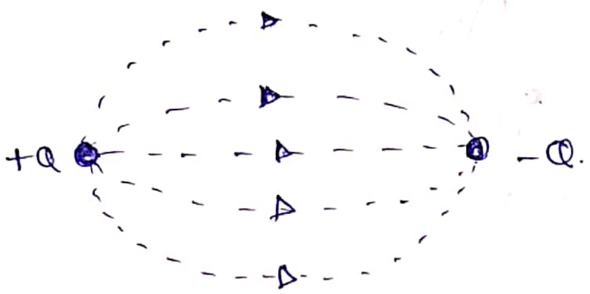
Electrictric Flux density :

→ The total number of lines of force in any particular electric field is called the electric flux (Ψ).

Unit of electric flux is Coulomb

→





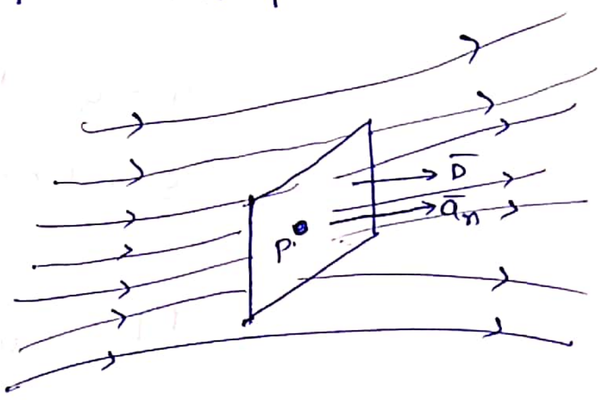
→ The net flux passing through the unit surface area is called the Electric flux Density ($\bar{D}$)

$$\boxed{\bar{D} = \frac{\Psi}{S}} \; C/m^2$$

where Ψ = Total flux

S = ~~To~~Total surface area of sphere.

Vector form of Electric flux Density :



The flux density $\bar{D}$ at point $\bar{P}$ can be represented in the vector form

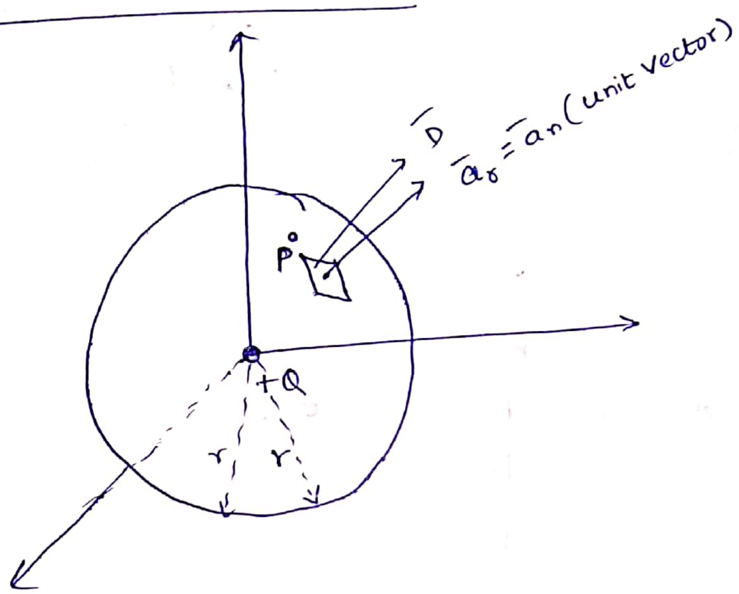
$$\boxed{\bar{D} = \frac{d\Psi}{ds} \hat{a}_n} \; C/m^2$$

where $d\psi$ → Total flux lines crossing normal through the differential surface area ds.

ds → Differential ~~area~~ surface area.

$\bar{a}_n$ → Unit vector in the direction normal to the differential surface area.

$\bar{D}$ due to point charge :-



→ Consider a point charge (+Q) placed at the centre of the imaginary sphere of radius r.

→ Flux lines originating from +Q are directed radially outward.

Magnitude of flux density at any point on the surface is

$$|\bar{D}| = \frac{\text{Total flux } \psi}{\text{Total surface area}}$$

$\psi = Q$ = Total flux

$S = 4\pi r^2$ = Total surface area

$$|\bar{D}| = \frac{Q}{4\pi r^2}$$

Unit Vector directed radially outwards & normal to the Surface at any point on the Surface is

$$\bar{a}_n = \bar{a}_r.$$

$$\boxed{\bar{D} = \frac{Q}{4\pi r^2}\,\bar{a}_r} \quad C/m^2$$

Relationship between $\bar{D}$ and $\bar{E}$:-

$\bar{E}$ at a distance 'r' from a charge $+Q$ is

$$\bar{E} = \frac{Q}{4\pi \varepsilon_0 r^2}\,\bar{a}_r$$

$$\frac{\bar{D}}{\bar{E}} = \frac{\frac{Q}{4\pi r^2}\,\bar{a}_r}{\frac{Q}{4\pi \varepsilon_0 r^2}\,\bar{a}_r} = \varepsilon_0$$

$$\Rightarrow \frac{\bar{D}}{\bar{E}} = \varepsilon_0 \Rightarrow \boxed{\bar{D} = \varepsilon_0 \bar{E}}$$ for free space.

$$\boxed{\bar{D} = \varepsilon \bar{E}}$$ for medium, other than free space

→ $\bar{D}$ & $\bar{E}$ both act in same direction

→ Relationship between $\bar{D}$ & $\bar{E}$ is applicable for any general Charge distribution.

→ $\bar{E}$ is a function of ε, but $\bar{D}$ is not.

Electric Flux Density for Various Charge Distributions :-

(1) Line charge:

Consider a line charge having uniform charge density of ρ_L c/m.

The total charge along the line is

$$Q = \int_L \rho_L \, dL$$

$$\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r = \cancel{\frac{\int \rho_L \, dl}{4\pi r^2} \bar{a}_r}$$

$$\boxed{\bar{D} = \frac{\int_L \rho_L \, dl}{4\pi r^2} \cdot \bar{a}_r}$$

For infinite line charge ; $\bar{E} = \dfrac{\rho_L}{2\pi \varepsilon_0 r} \bar{a}_r$

$$\bar{D} = \varepsilon_0 \bar{E} = \varepsilon_0 \cdot \frac{\rho_L}{2\pi \varepsilon_0 r} \bar{a}_r$$

$$\boxed{\bar{D} = \frac{\rho_L}{2\pi r} \bar{a}_r}$$

(2) Surface charge:

For a sheet of charge having uniform charge density of ρ_s c/m^2,

Total charge, $Q = \int_S \rho_s \cdot ds$.

$$\bar{D} = \frac{Q}{4\pi r^2} \bar{a}_r = \frac{\int_S \rho_s \cdot ds}{4\pi r^2} \cdot \bar{a}_r$$

If the sheet of charge is infinite, then

$$\bar{E} = \frac{\rho_s}{2\varepsilon_0} \bar{a}_n \; ; \quad \bar{D} = \varepsilon_0 \bar{E} = \varepsilon_0 \cdot \frac{\rho_s}{2\varepsilon_0} \cdot \bar{a}_n$$

$$\boxed{\bar{D} = \frac{\rho_s}{2} \bar{a}_n}$$

(3) Volume charge :

Consider a charge enclosed by a volume with a uniform charge density of ρ_v C/m^3.

$$Q = \int_v \rho_v \cdot dv \quad ; \quad \bar{E} = \frac{Q}{4\pi\epsilon_0 r^2}\,\bar{a}_r$$

$$\bar{D} = \frac{\int_v \rho_v \cdot dv}{4\pi r^2}\,\bar{a}_r$$

Gauss's Law:

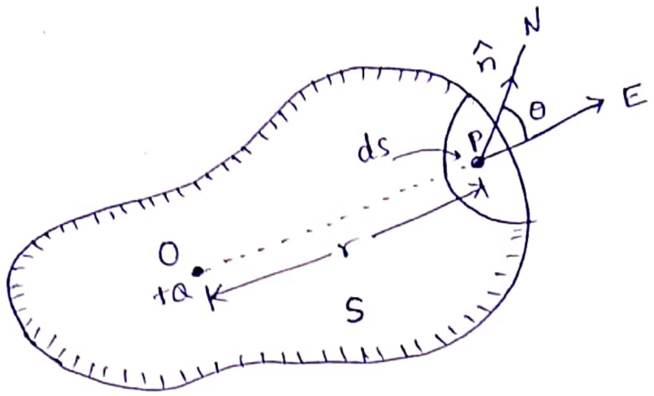
Statement:

"The electric flux passing through any closed surface is equal to the total charge enclosed by that Surface."

The electric field intensity at any point distant 'r' from the field-producing charge 'Q' is given by the equation

$$\bar{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{a}_r .$$

Imagine a closed surface enclosing the charge 'Q'



Fig(a): A closed surface 'S' enclosing a charge 'Q'.

where $\hat{n}$ - Unit normal to the surface element 'ds' around P.

PN - Normal drawn to the surface "ds" from P.

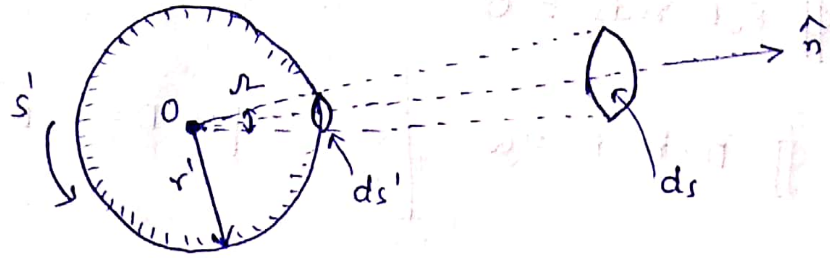
The surface integral of the normal component of the electric field $\bar{E}$ over the closed surface S enclosing Q is given by

$$\oint\!\!\oint_S \bar{E}\cdot\hat{n}\, ds = \frac{Q}{4\pi\varepsilon_0}\oint\!\!\oint_S \frac{a_r\cdot\hat{n}}{r^2}\, ds \longrightarrow (1)$$

where $a_r\cdot\hat{n}\, ds$ is the projection of ds on a plane ⊥r to the vector $\hat{r} = r\cdot\bar{a}_r$. This projected area divided by r^2 is the solid angle subtended by ds = say, $d\Omega$

In general

$$\Omega = \frac{A}{r^2} = \text{Solid Angle}$$



Fig(b): Spherical surface s', for evaluation of the Solid angle Subtended by ds,

→ Consider the surface area ds, each point on the boundary of which is jointed to O by a straight line as shown in fig(b); a slender Cone is then formed. Now Construct a sphere s' with O as Centre and r' as radius.

Solid angle Subtended by ds is equal to the Solid angle Subtended by ds' which is an element of the Spherical surface s' centred at O.

Therefore, for a closed Surface

$$\oiint_S \frac{a_r \cdot \hat{n}}{r^2} \cdot ds = \oiint_{s'} \frac{a_r \cdot \hat{n}}{(r')^2} \cdot ds'$$

$$= \frac{(\text{Surface area of the Sphere of radius } r')}{(r')^2}$$

$$= \frac{4\pi (r')^2}{(r')^2}$$

$$= 4\pi$$

Substitute the above results in eq (1)

$$\oint\!\!\oint_S E\cdot\hat{n}\,ds = \frac{Q}{4\pi\varepsilon_0}(4\pi) = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \oint\!\!\oint \varepsilon_0\bar{E}\cdot\hat{n}\,ds = Q$$

$$\Rightarrow \boxed{\oint\!\!\oint \bar{D}\cdot\hat{n}\cdot ds = Q} \qquad [\because D = \varepsilon_0 E]$$

" Surface integral of the normal component of the electric flux density vector $\bar{D}$ over any closed surface is equal to the total charge enclosed by that surface". This is known as Gauss's law.

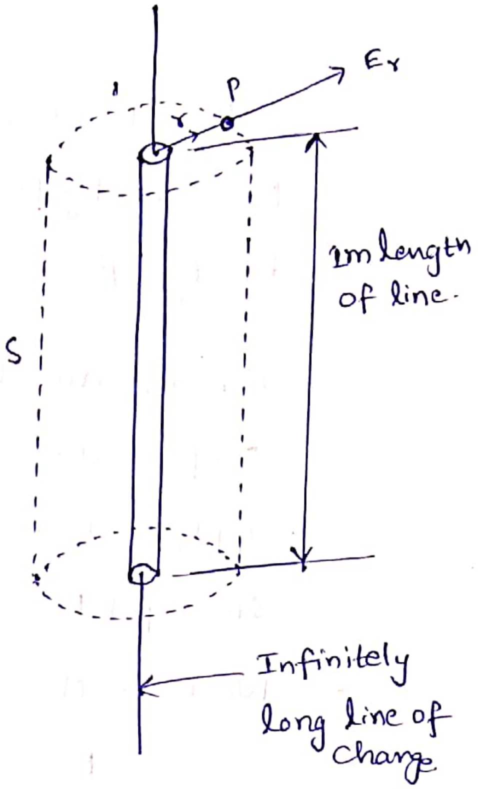
Gaussian Surface :

Gauss's law is very useful to find out electric field intensity. To find $\bar{E}$, we construct an imaginary surface called "Gaussian Surface".

The electric field must be uniform at every point on this surface. It must be normal to the surface considered.

Applications of Gauss's Law :-

(1) Determine the field at a distance 'r' from an infinite line Charge of strength $+\rho_L$ C/m length.



Sol:

S (Gaussian Surface) Co-axial Cylindrical Surface surrounding the charged line over a metre-length

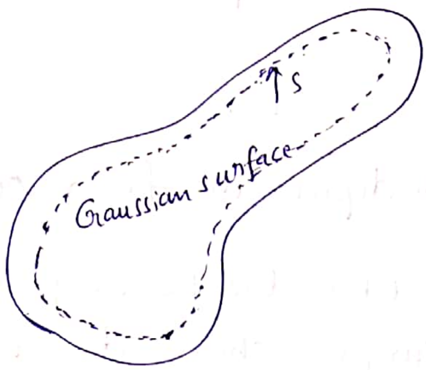
Apply Gauss's law for a metre-length of the charged line,

$$\iint_S E\cdot\hat{n}\,ds = (2\pi r)(1)\,E_r = \frac{\rho_L(1)}{\varepsilon_0}$$

$$\Rightarrow \boxed{E = E_r = \frac{\rho_L}{2\pi\varepsilon_0 r}}\ \bar{a}_r$$

NOTE

(i) What is the field inside the Charged Conductor ?

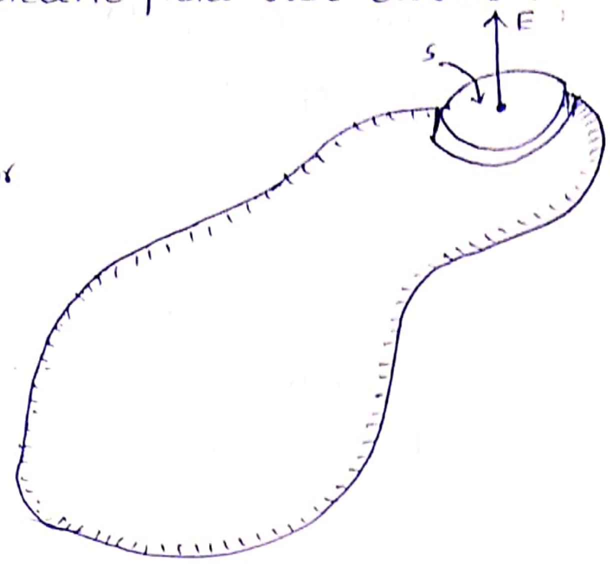


Sol: On the basis of Gauss's law, the net charge ~~enclosed~~ by ~~each of the Gaussian Surfaces is zero~~. of the charged conductor resides on its Surface.

So, the charge enclosed is zero.

so the electric field is zero.

(2) Consider a closed pill-box shaped surface 'S' resting on the surface of a charged conductor as shown in figure. Find an expression for the electric field just outside the conductor.



Sol:

Let the charge on the conductor be given by the surface charge density ρ_s.

→ Consider an elementary surface area 'ds', then applying Gauss's law to the small pill-box shaped surface 'S'.

→ We have the surface integral of the normal component of

$$\bar{E} = E \cdot \hat{n} \cdot ds = \frac{dQ}{\varepsilon_0} = \frac{\rho_s \cdot ds}{\varepsilon_0}$$

$$\Rightarrow \quad E \cdot \hat{n} \cdot \cancel{ds} = \frac{\rho_s \cdot \cancel{ds}}{\varepsilon_0}$$

$$\Rightarrow \quad \boxed{\bar{E} = \frac{\rho_s}{\varepsilon_0} \hat{n}}$$

which depicts the field outside a charged conductor.

(3) Find the EFI due to uniformly charged infinite plane sheet with surface charge density of "ρ_s" C/m^2.

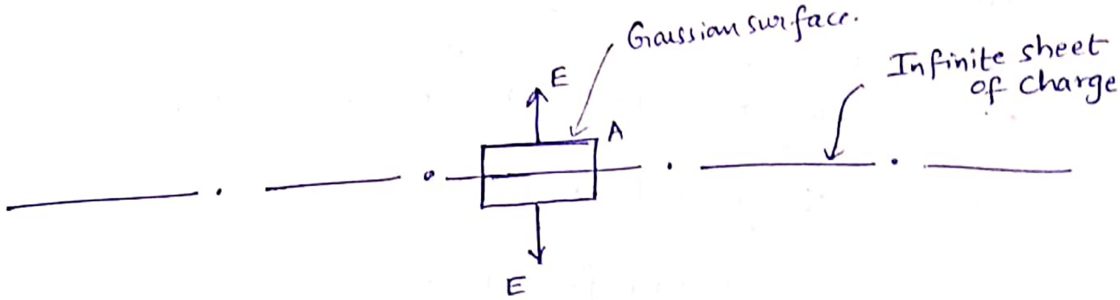


Fig: ~~Ele~~ Electric field due to infinite sheet of charge.

By Gauss's Law

$$\iint E\cdot\hat{n}\cdot ds = 2E\cancel{A} = \frac{\rho_s\cdot\cancel{A}}{\varepsilon_0}$$

Where A = Area of each end section

$$\Rightarrow \boxed{E = \frac{\rho_s}{2\varepsilon_0}}\cdot\hat{n}.$$

(4) Determine the Variation of field from point to point due to:

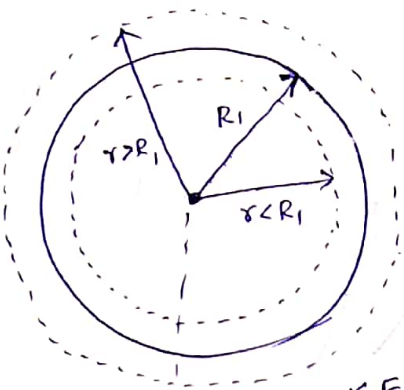
(i) a single Spherical shell of charge with radius R_1;

(ii) two Concentric Spherical shells of charge of radii R_1(inner) and R_2 (outer);

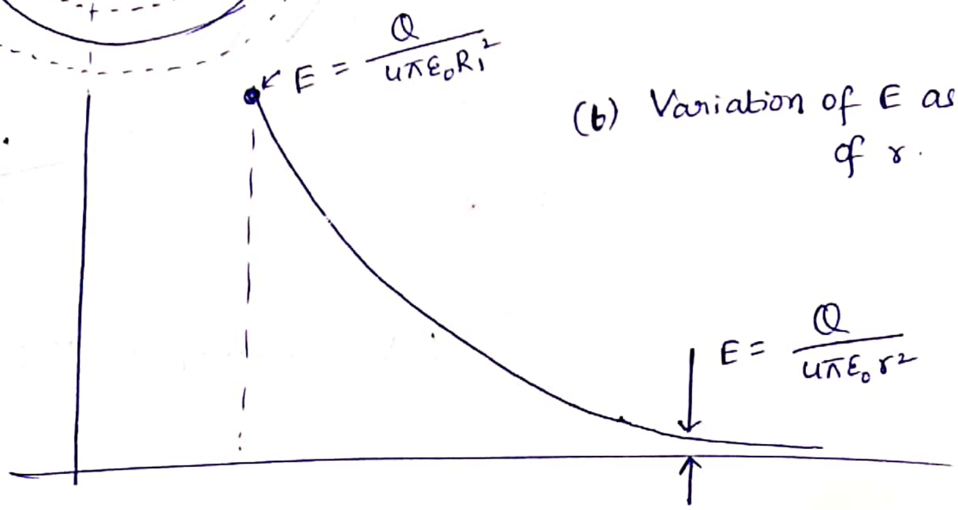
(iii) Spherical Volume distribution of charge of radius R and density ρ.

Sketch the results.

Sol: (i) Single Shell of charge:



(a) Shell of charge at $r = R_1$



(b) Variation of E as function of r.

At $r = R_1$, $E = \dfrac{Q}{4\pi \varepsilon_0 R_1^2}$

For $r < R_1 \Rightarrow \iint_S \bar{D}\cdot\hat{n}\cdot ds = \varepsilon_0 \iint E\cdot\hat{n}\cdot ds = 0$

Thus $\boxed{E = 0}$

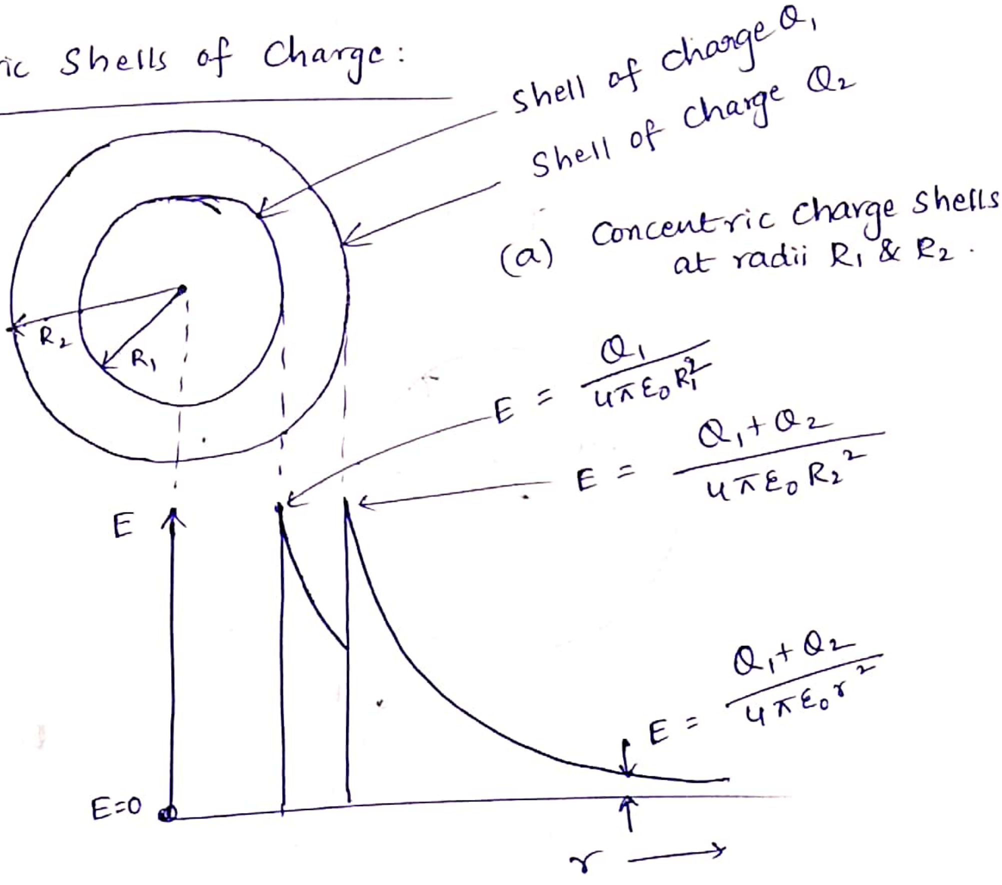
For $r > R_1 \Rightarrow \iint_S \bar{D}\cdot\hat{n}\cdot ds = Q.$

$$\iint_S \varepsilon_0 \bar{E}\cdot\hat{n}\cdot ds = Q$$

$$\varepsilon_0 \bar{E}\left[4\pi \cancel{R_0}^2\right] = Q$$

$$\Rightarrow \boxed{\bar{E} = \frac{Q}{4\pi\varepsilon_0 \cancel{R_0}^2}} \quad \bar{a}_r$$

(ii) Two Concentric Shells of Charge:



(a) Concentric charge shells at radii R_1 & R_2.

(b) Variation of $\bar{E}$ as function of r.

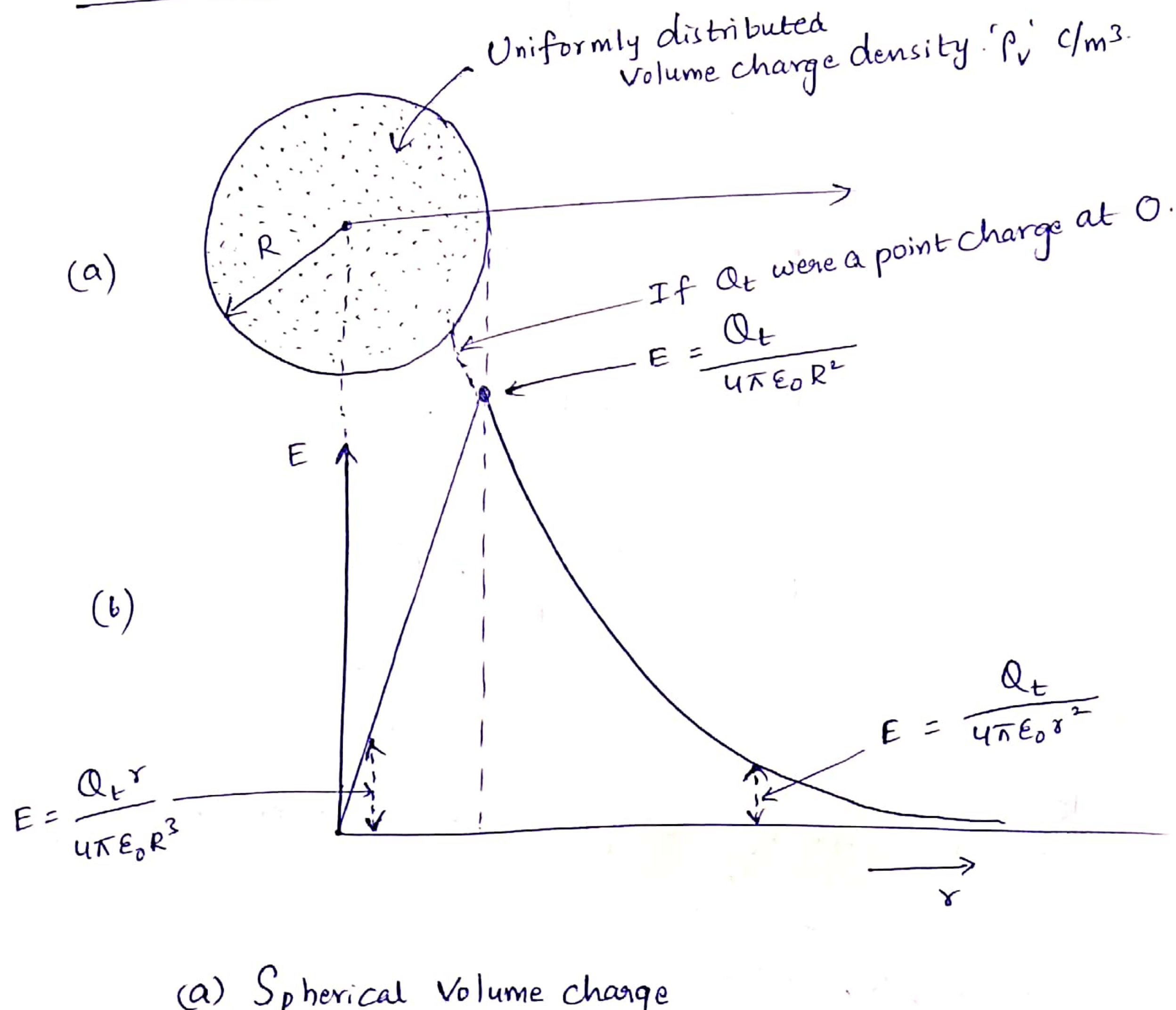
At R_1, $E = \dfrac{Q}{4\pi\varepsilon_0 R_1^2}\,\bar{a}_r$

For $R_1 \le r \le R_2$, $\bar{E} = \dfrac{Q_1}{4\pi\varepsilon_0 r^2}\,\bar{a}_r$

For $r = R_1 + dr$, $\bar{E} = \dfrac{Q_1}{4\pi\varepsilon_0 r^2}\,\bar{a}_r$

For $r = R_2 + dr$ (or) $r > R_2$, $\bar{E} = \dfrac{Q_1 + Q_2}{4\pi\varepsilon_0 r^2}\,\bar{a}_r$

(iii) Spherical Volume distribution of Charge:



(a) Spherical Volume charge

(b) Variation of E(r)

→ Consider any point with in the spherical volume of the charge i.e., for $r < R$.

→ As we have uniformly distributed charge over the entire volume, the charge contained by a concentric sphere of radius $r < R$ is proportional to the volume which is proportional to the cube of the radius.

→ So that the charge Q_r at any radius 'r' less than R is related to the total volume charge

$$\frac{Q_r}{Q_t} = \left[\frac{r}{R}\right]^3$$

$$Q_r = Q_t \left[\frac{r}{R}\right]^3$$

For $r < R$,
$$\bar{E} = \frac{Q_r}{4\pi\epsilon_0 r^2} \bar{a}_r$$

$$\bar{E} = \frac{Q_t \left[\frac{r^{\cancel{3}}}{R^3}\right]}{4\pi\epsilon_0 \cancel{r^2}} \cdot \bar{a}_r$$

$$\boxed{\bar{E} = \frac{Q_t \cdot r}{4\pi\epsilon_0 R^3} \cdot \bar{a}_r}$$

→ ~~For~~ At the centre of the sphere, $r = 0$. Then $\bar{E} = 0$.

For $r \geq R$,
$$\boxed{E = \frac{Q_t}{4\pi\epsilon_0 r^2} \bar{a}_r}$$